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FINAN 6510
FINANCIAL APPLICATIONS OF STATISTICAL MODELS
ASSIGNMENT # 2

Exercise 1. The probability of a new office building being built in town is 64%. The probability of a new office building that includes a coffee shop being built in town is 58%. If a new office building is built in town, what is the probability that it includes a coffee shop?

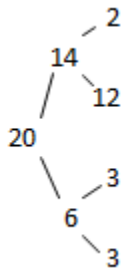
$P(NewBuilding_{CoffeeShop}) = 58\%$

$$\frac{P(New_{Building} \cap Coffee_{Shop})}{P(New_{Building})} = \frac{58\%}{1} = 58\%$$

Exercise 2. There is a 40% probability that the economy will be good next year and a 60% probability that it will be bad. If the economy is good, there is a 50% probability of a bull market, a 30% probability of a normal market, and a 20% probability of a bear market. If the economy is bad, there is a 20% probability of a bull market, a 30% probability of a normal market, and a 50% probability of a bear market. What is the joint probability of a bad economy and a bull market?

$P(Market_{Bull} \cap Economy_{Bad}) = P(M_B|E_B) * P(E_B) = .2 * .6 = .12\ or\ 12\%$

Exercise 3. An analyst has a list of twenty bonds of which fourteen are callable, and of which five have warrants attached to them. Two of the callable bonds have warrants attached to them. If a single bond is chosen at random, what is the probability that it is either a callable bond or a bond with a warrant (or both)?



$$P_{Callable} = \frac{14}{20} = .7$$

$$P_{Warrant} = \frac{5}{20} = .25$$

$$P(Callable_{Warrant}) = \frac{2}{20} = .1$$

$P(B_{Callable} \cup B_{Warrant}) = P(B_{Call}) + P(B_{Warr}) - P(B_{Call} \cap B_{Warr})$

$= .7 + .25 - .1 = .85$

Exercise 4. An options trader recently wrote two put options on two different underlying stocks (AlphaDog Software and OmegaWolf Publishing), both with a strike price of $12.25. The probabilities that the prices of AlphaDog and OmegaWolf stock will decline below the strike price are 89% and 32%, respectively, and these probabilities are independent. What is the probability that at least one of the put options will fall below the strike price?

$$P(at\ least\ one\ \alpha\ and\ \omega) = 1 - P(neither\ \alpha\ nor\ \omega) = 1 - P\big((\sim\alpha) \cap (\sim\omega)\big)$$

$$= 1 - P(\sim\alpha) * P(\sim\omega) = 1 - \big(1 - P(\alpha)\big)\big(1 - P(\omega)\big)$$

$$= 1 - (1 - .89)(1 - .32)$$

$$= .9252$$

Exercise 5. In a given portfolio, half of the stocks have a beta greater than one. Of those with a beta greater than one, a third are in a computer-related business. What is the probability of a randomly drawn stock from the portfolio having both a beta greater than one and being in a computer-related business?

$$P\big(Business_{Computer} \cap \beta_{>1}\big) = P(B_C|\beta_{>1}) * P(\beta_{>1}) = \frac{1}{3} * \frac{1}{2} = \frac{1}{6} or\ .167$$

Exercise 6. A bond portfolio consists of four BB-rated bonds. Each has a probability of default of 24%, and these probabilities are independent. What are the probabilities of all the bonds defaulting and of all the bonds not defaulting, respectively?

Probability of Default

$$P(B_1 \cap B_2 \cap B_3 \cap B_4) = P(B_1|B_2|B_3|B_4) = .24 * .24 * .24 * .24 = .00332$$

Probability of Non-Default

$$P(B_1 \cap B_2 \cap B_3 \cap B_4) = (P(1 - B_1) * P(1 - B_2) * P(1 - B_3) * P(1 - B_4)) =$$

$$(.76 * .76 * .76 * .76) = .3336$$

Exercise 7. Data shows that seventy-five out of one hundred tourists who visit New York City visit the Empire State Building. It rains or snows in New York City one day in five. What is the joint probability that a randomly chosen tourist visits the Empire State Building on a day when it neither rains nor snows?

$$P\big(Tourist_{Empire}\big) = .75 \qquad P(Rain\ or\ Snow) = .2 \qquad 1 - P(Rain\ or\ Snow) = .8$$

$$P\big(Tourist_{Empire}\big) * \big(1 - P(Rain\ or\ Snow)\big) = .75 * .8 = .6$$

Exercise 8. If the probability of both a new Wal-Mart and a new Wendy's being built next month is 68% and the probability of a new Wal-Mart being built is 85%, what is the probability of a new Wendy's being built if a new Wal-Mart is built?

$P(WM|Wendy's) = .68 = P(WM) * P(Wendy's) = .68$

$= .85 * P(Wendy's) = .68$

$P(Wendy's) = \frac{.68}{.85} = .8$

Exercise 9. The probability of each of three independent events is shown in the table below:

Event	Probability of Occurrence
A	25%
B	15%
C	42%

What is the probability of A and C occurring, but not B?

$P(A) * P(C) * \big(1 - P(B)\big) = .25 * .42 * (1 - .15) = .0893$

Exercise 10. A parking lot has one hundred red and blue cars in it. 35% of the cars are red. 80% of the red cars have radios. 60% of the blue cars have radios. What is the probability of selecting a car at random that is either red or has a radio (or both)?

$P(Red \cup Radio) = P(Red) + P\big(Radio \cap (\sim Red)\big)$

$= P(Red) + P\big(Radio|(\sim Red)\big)P(\sim Red)$

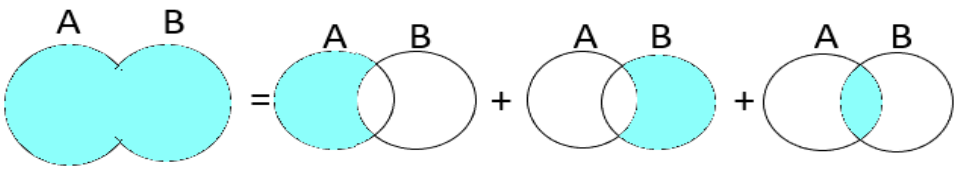
$= .35 + .6 * .65 = .74$

Exercise 11 (BONUS). Prove that, for any events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Ω is the set of all possible outcomes.

If $\Omega = A \cup B \; and \; if \; x \in A \cup B$, then $x \in A/B \; or \; x \in B/A \; or \; x \in A \cap B$ shown below:

Ω



$A/B = A - A \cap B$ and $B/A = B - A \cap B$

Then $P(A \cup B) = P(A \,/\, B) + P(B \,/\, A) + P(A \cap B)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) + P(A \cap B)$

= P(A) + P(B) – 2*P($A \cap B$) + P($A \cap B$)

$= P(A) + P(B) - P(A \cap B)$ ***as intended*** ☒ The reverse direction is similar