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JREPM

Industrial Real Estate Cycles: Markov Chain Applications

Richard D. Evans
Andrew G. Mueller

Executive Summary. Adding a stochastic element to a well-understood real estate cycle model offers opportunities like those seen in earlier such syntheses of real estate analysis and statistics. The discrete real estate cycle points in the model require a discrete probability model, here a first order Markov chain. Many statistical applications flow from the combined model. Three Markov chain count variables have obvious real estate cycle appeal. Staying time, first recurrence time, and first passage time already exist in the Markov chain literature but only staying time is in the real estate cycle literature. The most fundamental innovation is in probabilistic forecasting. Being able to describe real estate cycle risk, cycle point by cycle point many quarters ahead, could improve evaluation of prospects for property disposal. It is also a simple spreadsheet application to describe real estate cycle risks that influence cash flows from operations across four-quarter spans.

Investors' need for real estate cycle forecasts come from the cycle's importance for cash flow anticipations that drive valuation mechanics, investment analysis, and many other applications, such as decisions to sell versus hold. An analyst with strong econometric skills and expensive data sets on the drivers of supply and demand may generate cost-effective forecasts four to eight quarters into the future. Longer term real estate cycle forecasts often have such wide prediction intervals that little mention is made of the uncertainty that grows quickly, even with the best of forecasting methods. Forecasting the real estate cycle conditions that will drive profitability three to ten years ahead often becomes an effort to articulate assumptions for a discounted cash flow model that are so "plausible" that no reader could reliably dispute them. The strongest investment analyst may not seem more sophisticated than one that assumes that cycle conditions will stay at currently existing levels, or that variables such as occupancy will trend over the years from current levels to the market area's long-term average level.

The analysis applied here uses a real estate cycle model that has become widely accepted and understood over the last 20 years as the base for adding a stochastic element to describe uncertainty. This sort of synthesis has paid high dividends for real estate researchers. For decades, appraisers and other real

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estate analysts understood principles of "contribution" of the elements of comparison and units of comparison in building widely-accepted, accurate estimates of the value of real property. When a formal stochastic element was added to the existing principles, real estate analysts exploited the fertile applications of hedonic pricing models (Rosen, 1974). Even earlier, the principles of price index construction were well established. Then a leap in applicability was achieved by adding a stochastic element to describe uncertainty through repeat sale price indexes (Bailey, Muth, and Nourse, 1963).

Both of these epic improvements in the research methods applied to real estate involved a continuous variable, price, and an appropriate, continuous random error term that allowed the exploitation of a whole library of regression models. The role of the stochastic element was not to "explain" price, but to characterize uncertainty in a way that allowed existing statistical methods to combine with existing, established real estate models. Much of the empirical innovation has come from adjusting the description of the uncertainty.

The base real estate cycle model used here is Mueller's (1995) model, which is widely accepted. That model includes 16 discrete cycle points, briefly described below. A discrete probability model—a Markov chain model—is added to the existing real estate cycle model. Just as the error term in a regression model of real estate prices seems only of arcane interest in terms of patterns over time and location and across observations without the contribution of the rest of the model, only this real estate cycle model makes the stochastic model meaningful.

The stochastic element added here is specified as a first order, regular, ergodic Markov chain. Evans and Mueller (forthcoming) specify commercial real estate cycles as Markov chains. The increased sample size available with the passage of time allows us to confirm the specification of the industrial markets' Markov chain as a first order model. We omit that discussion to save space.

Perhaps the most fundamental contribution of the Markov chain model of industrial real estate cycles is in forecasting. The numerical examples here include two uses of probabilistic forecasts for real estate cycle conditions (here quarter-by-quarter for 20 quarters) given an initial status of a city's industrial real estate market. A first use of these forecasts would be to anticipate cycle risks if a real estate analyst anticipates that a property acquired now would be resold, for example, after a five-year holding period. A second use gives the relative frequency of alternative cycle conditions over each four-quarter span of the holding period. An analyst could describe the cycle risk characteristics of an industrial investment's annual cash flows from operations.

The Markov chain forecasts are not "what will happen" or "why cycle conditions evolve." The probabilistic forecasts are about what cycle points are not possible or are very unlikely to occur in defined future periods, while other cycle conditions have higher measured probabilities. An expert econometrician who has acquired an adequate database of supply and demand factors could explain why cycle conditions did or will change in predicted ways. The numerical examples of the Markov chain forecasting show that a practitioner with few econometric skills and a very limited information set may use common spreadsheet functions to generate a description of risks across real estate cycle points.

In addition to the forecasting applications, three long established Markov chain "count variables" have intuitive importance to real estate cycle analysts who are willing to add a stochastic element to models. First, the number of quarters that a city market for industrial real estate will take to make its first change in cycle conditions, given just that the initial cycle point is known, is called "staying time," or "holding time." The mean and variance of staying time are easy calculations from spreadsheets used to specify Markov chains. An individual element applicable to one cycle point determines both the mean and variance of staying time for that cycle point. A simple, first order Markov chain can show the commonly-perceived action of real estate cycles slowing down, pausing, and speeding up. New to this analysis is a discussion about mean staying time's weakness in describing central tendencies, common for distributions of all three real estate cycle count variables.

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As a new application for the real estate cycle literature, in this paper we deal with the time taken for an industrial real market in some known cycle point to first return to the same cycle conditions. *First Recurrence Time* is a random variable. Mean *First Recurrence Times* are simply calculated as the reciprocal of elements of the Markov chain's "stationary vector" of probabilities. In addition, this unique vector may offer an intuitive interpretation in very long-term anticipations. The standard spreadsheet estimates reported here for these probabilities and the mean first recurrence are plausible.

Third, given that a city market is in a particular real estate cycle point, the number of quarters that it will take to first move to another specified cycle point is a random variable, *First Passage Time*. For example, suppose that a city's industrial real estate market is recovering, but just out of recession. How many quarters will it take the market to move to the cycle point that has rent and occupancy conditions that make new construction financially feasible? Information on this first passage time would be useful to developers who want to time their construction steps so as to not bring new property to market before this profitable environment, but also not being slow to exploit opportunities. Mean *First Passage Time* is intuitively important to real estate developers and investors.

Mean *First Passage Time* also may be calculated using matrix functions in spreadsheet programs, applying calculations that were established more than 50 years ago. However, the spreadsheet results (not shown here) include some "unrealistic" mean *First Passage Time* calculations, and some that are impossible. Using special matrix methods allows, for the first time, plausible results for mean *First Passage Time* in a real estate cycle application of Markov chain analysis. These are the only calculations here that are not part of a standard spreadsheet program.

All three of the intuitively appealing count variables are either non-negative or strictly positive. That limit on the distributions and other factors discussed here bring positive skewness to the distributions. Detailed study of the count variables indicates that means are not the best measure of central tendency for these variables. Experienced real estate cycle analysts will have more confidence in describing these distributions with the median instead of the mean.

Real Estate Cycle Models and Data

In this commercial real estate application, the discrete "Markov states" are the 16 real estate cycle points defined by Mueller (1995). Cycle Point 11 (CP11) is perhaps the most profitable set of cycle conditions. Occupancy is at historic highs for the given city market. Rents are high and growing. Supply and demand grow in equilibrium. Because demand fluctuates and because supply increases come from construction commitments set in motion many periods in the past, it eventually occurs that less profitable cycle conditions evolve.

CP 12 is still a highly profitable set of conditions, but hyper-supply has developed because new construction deliveries have outpaced growth in demand. Hyper-supply is more severe at CP13, where occupancy is still higher than long-term city averages, but rent growth is slower than at CP12. The hyper-supply stage ends when CP14 marks the fall of occupancy to levels that are just average for the city market.

CP 14 marks the transition of the city market from hyper-supply to recession. Ill-timed construction completions still come to market in CPs 15 and 16, but demand is not adequate. Recession brings low and decreasing occupancy, as well as rent growth that is not keeping pace with inflation, which is often negative.

Mueller's (1995) cycle taxonomy lists CP1 as the trough of recession, but also the transition step to recovery. Occupancy is at historic lows for the city market, and rents are low and not growing. At least, completions are no longer being added to supply. The first state of recovery comes with CP2. Occupancy improves, but rental growth is still low or not growing. CPs 3, 4, and 5 are also recovery environments. New construction is still not feasible, but occupancy improves, as does rent growth.

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CP 6 marks occupancy returning to the city's long-term average. That cycle point serves as the transition from recovery to expansion. CP 7 in the expansion stage of the real estate cycle has above average occupancy and stronger rent growth. New construction is feasible again at the next cycle point, CP8. It has occupancy high enough and rents high enough that financial sources will now support new construction. Profitability builds at other expansion CPs, 9 and 10. CP 11 has the highest occupancy and very strong rent levels and growth.

Once in CP11, demand growth may outstrip supply growth, causing the commercial real estate cycle conditions to not "move ever forward," as the previous discussion may imply. Supply growth is limited by decisions made many periods before, while demand may have a growth spurt as part of a long period of growth. Thus, it is easy to see how real estate cycles may transition from a point such as CP11 to CP10 instead of "only forward" in the real estate cycle taxonomy. Also, changes of more than "one step" in the array of 16 cycle points often are noted in Mueller's *Cycle Monitor* quarterly reports.

Mueller's *Cycle Monitor* is published by Dividend Capital, with downloads of current and past quarterly issues provided without charge [link tested April 8, 2016: Dividend Capital Research, *Real Estate Market Cycle Monitor*: http://www.dividendcapital.com/]. Cycle conditions for each of more than 50 city markets for industrial properties are in the quarterly *Cycle Monitor*. The format of the report allows an essential tabulation for empirical Markov chain analysis, a tally of the frequency of change from each possible cycle point to each other cycle point, shown at the top of Exhibit 1. The sample period for this study is from 1996:Q4 to 2014:2, with some editing.

Brief Introduction to Markov Chain Models, with an Empirical Application to Real Estate Cycles

A first order Markov chain is a stochastic process that describes discrete states as being random over time, with the state that exists in the first step ahead depending on only the initial state that exists currently (or may exist initially, in some applications) and a set of transition probabilities across states period-to-period. In this commercial real estate application, the "states" are 16 real estate cycle points defined by Mueller (1995).

Describing algebraic manipulations without using matrix algebra would often be overwhelmingly complex, just as not using spreadsheet matrix methods would be impractical. Calculations reported in this section are done with spreadsheet matrix functions.

In spreadsheet or algebraic representation, we use p^k as a vector of the probabilities that each of the 16 cycle points exists in a given city at a time period k steps ahead:

$$p^k = (p_1^k p_2^k p_3^k p_4^k p_5^k p_6^k p_7^k p_8^k p_9^k p_{10}^k p_{11}^k p_{12}^k p_{13}^k p_{14}^k p_{15}^k p_{16}^k).$$

This is a format for forecasting k steps ahead—not what will happen, but what cannot happen (indicated by elements that are zeroes) and what may happen (with positive indicated likelihoods).

We use p^0 to describe initial cycle probabilities—zero steps ahead, the current time period. Consistently in this application, all the elements of p^0 are zero, except for one cycle point that has a 100% probability to indicate that we know it with certainty, $p^0 = (0\ 0\ 0 \ldots 1\ 0\ 0\ 0)$.

The set of transition probabilities from one state to another, P, may be generated by other probability models, from theoretical models of the system being studied, from subjective judgment and common sense, or from historical data, as done here. All of these are valid in the proper context. The array may be written in general as:

$$P = \begin{bmatrix} p_{1,1}\ p_{1,2}\ p_{1,3} \cdots p_{1,16} \\ p_{2,1}\ p_{2,2}\ p_{2,3} \cdots p_{2,16} \\ p_{3,1}\ p_{3,2}\ p_{3,3} \cdots p_{3,16} \\ \cdots \\ p_{16,1}\ p_{16,2}\ p_{16,3} \cdots p_{16,16} \end{bmatrix},$$

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Exhibit 1 | Industrial Real Estate Cycles Changes and Estimates of Transition Probabilities: 1996:Q4–2014:Q2

Panel A: Tally Matrix: Industrial Market Cycle Changes

Cycle Point in an Initial Quarter	Cycle Point in Following Quarter: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Sum Row
1	739	97	10	0	1	0	0	0	0	0	0	0	0	0	0	1	848
2	21	349	74	11	3	2	0	0	0	0	0	0	0	0	0	12	472
3	0	9	261	66	4	4	0	0	0	0	0	0	0	0	9	0	353
4	0	0	6	173	42	19	0	0	0	0	0	0	0	0	15	0	255
5	0	0	0	4	71	28	10	1	0	0	0	0	0	0	5	0	119
6	0	0	0	1	5	104	32	7	1	0	0	0	0	0	4	0	154
7	0	0	0	0	0	3	63	32	2	2	0	0	6	2	0	0	110
8	0	0	0	0	0	0	4	85	31	5	2	3	0	0	0	0	130
9	0	0	0	0	0	0	4	8	176	31	6	16	2	0	0	0	243
10	0	0	0	0	0	0	0	0	15	130	39	11	2	0	0	0	197
11	0	0	0	0	0	0	0	0	5	11	180	61	4	1	0	0	262
12	0	0	0	0	0	0	0	0	1	10	27	87	53	11	7	0	196
13	1	1	2	0	0	0	0	0	2	4	6	11	42	20	19	2	110
14	2	5	2	0	0	0	0	0	0	0	0	1	1	23	17	7	58
15	7	5	1	0	0	0	0	0	0	0	1	0	0	1	72	63	150
16	78	7	0	0	0	0	0	0	0	0	0	0	0	0	2	36	123

Panel B: Matrix P: Industrial Market Cycle Transition Probability Estimates

Cycle Point in an Initial Quarter	Cycle Point in Following Quarter: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0.871	0.114	0.012	0	0.001	0	0	0	0	0	0	0	0	0	0	0.001
2	0.044	0.739	0.157	0.023	0.006	0.004	0	0	0	0	0	0	0	0	0	0.025
3	0	0.025	0.739	0.187	0.011	0.011	0	0	0	0	0	0	0	0	0.025	0
4	0	0	0.024	0.678	0.165	0.075	0	0	0	0	0	0	0	0	0.059	0
5	0	0	0	0.034	0.597	0.235	0.084	0.008	0	0	0	0	0	0	0.042	0
6	0	0	0	0.006	0.032	0.675	0.208	0.045	0.006	0	0	0	0	0	0.026	0
7	0	0	0	0	0	0.027	0.573	0.291	0.018	0.018	0	0	0.055	0.018	0	0
8	0	0	0	0	0	0	0.031	0.654	0.238	0.038	0.015	0.023	0	0	0	0
9	0	0	0	0	0	0	0.016	0.033	0.724	0.128	0.025	0.066	0.008	0	0	0
10	0	0	0	0	0	0	0	0	0.076	0.660	0.198	0.056	0.010	0	0	0
11	0	0	0	0	0	0	0	0	0.019	0.042	0.687	0.233	0.015	0.004	0	0
12	0	0	0	0	0	0	0	0	0.005	0.051	0.138	0.444	0.270	0.056	0.036	0
13	0.009	0.009	0.018	0	0	0	0	0	0.018	0.036	0.055	0.100	0.382	0.182	0.173	0.018
14	0.034	0.086	0.034	0	0	0	0	0	0	0	0	0.017	0.017	0.397	0.293	0.121
15	0.047	0.033	0.007	0	0	0	0	0	0	0	0.007	0	0	0.007	0.480	0.420
16	0.634	0.057	0	0	0	0	0	0	0	0	0	0	0	0	0.016	0.293

Notes: In Panel A, the entry "97" in row 1, at column 2 means that 97 observations occurred for transitions from cycle point 1 to cycle point 2 during the sample period. In Panel B, the entry "0.114" in row 1, at column 2 means that 97/848 rounds to this estimated probability for transitions from cycle point 1 to cycle point 2 during the sample period. "0" means exactly zero, while "0.000" means that the probability rounds to that expression.

where p_{ij} is the probability of a one-quarter change from cycle point i to cycle point j. The matrix for the full Mueller taxonomy has 256 elements, (16)(16).

Exhibit 1 shows an empirical development of P, the transition matrix. The tally matrix shows tabulations of industrial markets' cycle point changes as the frequencies of transitions from one cycle point to alternative cycle points. The relative frequencies of these changes estimate the transition probabilities at the bottom of Exhibit 1. For example, 848 cases occurred in the sample period for which a city market was originally in CP1. Of these, there are 97

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occurrences of a change from CP1 to CP2. The 97/848 ≈ 0.114 relative frequency is a maximum likelihood estimate of the probability of transition in that direction between the two cycle points.

A Markov chain model for industrial markets has "no absorbing states." This conclusion comes from examining the estimated P matrix in Exhibit 1, noting that there is no cycle point with a property such that, once in that state, there is 1.00 probability of staying in that state. Also, the estimated Markov chain here is "ergodic," meaning that it is possible to go from any cycle point to any other cycle point in some finite number of steps in the stochastic process. Sufficient conditions for proving this are met by raising the estimated P matrix to higher and higher powers, finding some power, k, such that P^k has no zero elements. In this case, there are no zero elements in P^4. This estimated Markov chain model is "regular." Proof of this property is in the ability to calculate the stochastic process's "stationary vector," as discussed in a section below dealing with "first recurrence time."

The industrial market cycle in this example is not an example of a "cyclical" Markov chain stochastic process. To be a cyclical Markov chain, there must be just one route into each state of the Markov process. An example would be:

$$P = \begin{bmatrix} .5 & .5 & 0 & 0 & 0 & \dots & 0 \\ 0 & .5 & .5 & 0 & 0 & \dots & 0 \\ 0 & 0 & .5 & .5 & 0 & \dots & 0 \\ 0 & 0 & 0 & .5 & .5 & \dots & 0 \\ . & . & . & . & . & \dots & . \\ 0 & 0 & 0 & 0 & 0 & .5 & .5 \\ .5 & 0 & 0 & 0 & 0 & 0 & .5 \end{bmatrix},$$

where the only way into state 2 is to have been in state 1 in the period immediately before. In Exhibit 1, the estimate of P shows that there is no state in the industrial real estate cycle that has this property.

It is important to note that the real estate cycle's Markov chain is not a cyclical Markov chain. This property causes potentially valuable count variables, *First Recurrence Time*, and *First Passage Time*, to be even more positively skewed random variables than if the model was cyclical. The means of the count variables are pulled away from the middle of the distribution by the non-trivial probabilities of given cycle points being "skipped" in one or more loops through the whole cycle. This generates very large values of the count variables that have non-zero probabilities. More discussion of this appears in the sections dealing with the count variables.

Probabilistic Forecast Applications

Ability to predict the probability of alternative states several periods ahead is one of the major applications of Markov chain modeling. The property of being a first order Markov chain means that $p^1 = p^0P$, that the real estate cycle point that occurs one-step ahead is a random variable with calculated probabilities given by the elements of p^1. Those probabilities depend on p^0 and P. Likewise, $p^2 = p^1P$, and, in general, $p^k = p^{k-1}P$. Alternatively, for any arbitrary number of steps ahead, $p^k = p^0P^k$, where P^k is the matrix P raised to the k power. These calculations come from knowing the spreadsheet function for matrix multiplication, or alternatively from using specialized mathematical software.

Exhibit 2 shows two numerical examples of generating probabilistic forecasts from alternative descriptions of initial conditions and the transition matrix, P (shown in Exhibit 1). First assuming that CP6 is known with certainty to be the initial condition of an industrial real estate market, p^0 is a row vector of zero probabilities, except for the value one entered as the sixth element. Using a spreadsheet's matrix multiplication function to multiply p^0 by the transition matrix P generates Exhibit 2's p^1, the one-step ahead forecast from the Markov chain model. Note that a one-step ahead forecast from an initial cycle point that is known with certainty always has the same elements as the corresponding row in the transition matrix.

As Exhibit 2's p^1 shows, when CP6 is certain in an initial period, CP6 is not certain in the next quarter. The probability of those same cycle conditions is calculated to fall to 0.675, not a certainty, but still the

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Exhibit 2 | Industrial Real Estate Cycle Point Probabilities across 20 Future Quarters

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Panel A: Initial Cycle Point 6 is Certain																
p^0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
p^1	0	0	0	0.006	0.032	0.675	0.208	0.045	0.006	0	0	0	0	0	0.026	0
p^2	0.001	0.001	0.000	0.010	0.042	0.470	0.264	0.121	0.024	0.006	0.001	0.001	0.011	0.004	0.032	0.011
p^3	0.010	0.003	0.001	0.011	0.042	0.335	0.256	0.179	0.055	0.017	0.005	0.007	0.019	0.009	0.033	0.017
p^4	0.022	0.006	0.003	0.012	0.038	0.244	0.226	0.209	0.091	0.031	0.013	0.015	0.024	0.012	0.034	0.020
p^5, p^6, p^7 omitted to save space																
p^8	0.069	0.027	0.013	0.011	0.019	0.080	0.107	0.176	0.180	0.087	0.066	0.056	0.035	0.018	0.033	0.024
p^9, p^{10}, p^{11} omitted to save space																
p^{12}	0.106	0.049	0.028	0.016	0.012	0.033	0.050	0.103	0.169	0.108	0.111	0.084	0.046	0.023	0.037	0.027
p^{13}, p^{14}, p^{15} omitted to save space																
p^{16}	0.141	0.069	0.043	0.024	0.012	0.021	0.028	0.059	0.129	0.102	0.127	0.092	0.050	0.025	0.043	0.032
p^{17}, p^{18}, p^{19} omitted to save space																
p^{20}	0.175	0.089	0.059	0.034	0.016	0.022	0.021	0.038	0.096	0.086	0.122	0.086	0.048	0.025	0.046	0.036
Panel B: Cycle Point Expected Relative Frequency across Four-Quarter Spans by Year																
Year 1	0.008	0.003	0.001	0.010	0.039	0.431	0.238	0.138	0.044	0.014	0.005	0.006	0.014	0.006	0.031	0.012
Year 2	0.052	0.019	0.008	0.011	0.025	0.125	0.147	0.199	0.156	0.068	0.045	0.041	0.031	0.016	0.033	0.023
Year 3	0.093	0.041	0.022	0.014	0.014	0.046	0.068	0.128	0.178	0.103	0.096	0.075	0.042	0.021	0.035	0.026
Year 4	0.128	0.062	0.037	0.021	0.012	0.024	0.035	0.073	0.144	0.106	0.123	0.090	0.049	0.025	0.041	0.030
Year 5	0.163	0.082	0.053	0.031	0.015	0.021	0.023	0.045	0.108	0.092	0.125	0.089	0.049	0.025	0.045	0.035
Panel C: Initial Cycle Point 11 Is Certain																
p^0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
p^1	0	0	0	0	0	0	0	0	0.019	0.042	0.687	0.233	0.015	0.004	0	0
p^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.032	0.071	0.514	0.268	0.080	0.020	0.012	0.001
p^3	0.003	0.003	0.003	0.000	0.000	0.000	0.001	0.002	0.041	0.089	0.409	0.253	0.112	0.040	0.035	0.009
p^4	0.012	0.009	0.006	0.001	0.000	0.000	0.001	0.003	0.048	0.098	0.341	0.227	0.120	0.052	0.057	0.024
p^5, p^6, p^7 omitted to save space																
p^8	0.126	0.051	0.027	0.009	0.002	0.002	0.003	0.006	0.057	0.093	0.206	0.144	0.086	0.049	0.078	0.060
p^9, p^{10}, p^{11} omitted to save space																
p^{12}	0.221	0.099	0.055	0.025	0.009	0.009	0.006	0.009	0.051	0.073	0.142	0.099	0.058	0.033	0.060	0.051
p^{13}, p^{14}, p^{15} omitted to save space																
p^{16}	0.254	0.128	0.080	0.044	0.019	0.020	0.012	0.013	0.044	0.056	0.103	0.072	0.042	0.024	0.049	0.041
p^{17}, p^{18}, p^{19} omitted to save space																
p^{20}	0.258	0.139	0.097	0.060	0.028	0.032	0.019	0.019	0.040	0.045	0.078	0.055	0.032	0.018	0.043	0.036
Panel D: Cycle Point Expected Relative Frequency across Four-Quarter Spans by Year																
Year 1	0.004	0.003	0.002	0.000	0.000	0.000	0.001	0.001	0.035	0.075	0.488	0.245	0.082	0.029	0.026	0.009
Year 2	0.078	0.034	0.018	0.005	0.001	0.001	0.002	0.005	0.056	0.098	0.247	0.172	0.100	0.054	0.078	0.052
Year 3	0.191	0.082	0.044	0.019	0.006	0.006	0.005	0.008	0.054	0.080	0.164	0.114	0.067	0.039	0.067	0.055
Year 4	0.245	0.118	0.071	0.037	0.015	0.015	0.009	0.011	0.046	0.062	0.117	0.081	0.047	0.027	0.052	0.045
Year 5	0.258	0.136	0.091	0.055	0.025	0.027	0.016	0.017	0.041	0.049	0.087	0.061	0.035	0.020	0.045	0.038

Notes: Matrix function for p^1: = MMULT($W100:$AL100,$W$83:$AL$98); for p^2: = MMULT($W101:$AL101,$W$83:$AL$98), where P is fixed in the array $W$83:$AL$98, p^0 is in array $W100:$AL100, and p^1 is in array $W101:$AL101. "0" means exactly zero, while "0.000" means that the probability rounds to that expression.

most likely cycle point after one period of change. In the p^1 forecast, CP7 has a 0.208 probability, while CP8 has a 0.045 probability in the prediction. Nine cycle points have the same zero probability as in the initial period. If a forecasted probability is zero, the associated cycle point is being described as not possible in that one period. These are large cycle jumps from CP6 that never occurred over only one quarter in the sample period, so the model gives them no chance of occurring in the forecasted period. Thus a

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Markov chain forecast is not "what" will happen, and certainly not "why." The forecast is probabilistic—what cannot happen, what may happen, and what is more likely than other possibilities.

As more periods are added to the forecast horizon, all the probabilities evolve. Repeating the matrix multiplication function using the p^1 just generated, $p^2 = p^1P$. CP6 is still the most likely condition two steps ahead, but its probability has evolved from 1, to 0.675, and to 0.470. Thus, cycle conditions are so changeable that after only two quarters, the probability of CP6 is less than the probability of "not CP6." Also, note that none of the p^2 probabilities are exactly zero, meaning that any cycle point could occur after only two quarters.

Copying the spreadsheet's matrix formula allows easy probabilistic forecasts for as many steps ahead as the real estate analyst wishes, 20 in the numerical example in Exhibit 2. Note that CP6 has only a 0.022 probability of being the industrial market cycle condition that would prevail if the analyst planned to sell the property after five years of ownership.

Exhibit 2 has a second numerical example, based on initial cycle conditions being CP11, supply and demand equilibrium, instead of CP6. General descriptions of the patterns into the future are qualitatively the same for the two examples. Note that the probability of being in CP11 is 0.678 one quarter ahead, 0.516 two ahead, and 0.409 three quarters ahead. These are slightly higher than seen for CP6 over the same forecast horizons. Also, note that these probabilities are not the same as staying in an initial cycle point for consecutive quarters of different durations. Those probabilities would be "staying times," discussed below. The occurrence of an industrial market starting in CP11 and being there three quarters later could occur in a variety of possible sequences of steps, with the sequence of "never changing" being one of particular interest.

Note that the 0.032 probability of CP6 conditions existing 20 steps ahead when a process began in CP11 is not much different than (the 0.022) when the process began at CP6. Finally, if the analyst is modeling the purchase of an industrial property that is in a market with CP11 conditions, followed by a sale of the property after a five-year holding period, then the probability of CP11 is 0.078—three other cycle points are more likely than supply and demand equilibrium. CP1's p^{20} probability of 0.258 makes it the most likely environment—selling the property in a trough of recession.

The analyst may also study the cycle risks associated with cash flows from operations from the numerical example in Exhibit 2. The elements of any p^k give estimates of the probability that alternative cycle points will prevail at exactly k steps ahead. An alternative interpretation of any probability is as a relative frequency. Averaging across any span of steps ahead, we can generate a relative frequency for industrial property to be operating in alternative cycle conditions during quarters within that span. The most common spans would be for the quarters 1–4, 5–8, 9–12, and so on, (for the first, second, third year, and so on), giving us annual descriptions of the operating conditions evolving over an investment holding period. Exhibit 2 shows that initial cycle conditions do influence both one-period and multi-period calculations, but that after only a few years, other cycle points come to dominate an analyst's anticipations.

Staying Time

Given an initial real estate cycle point for a city's industrial market, Exhibit 1 shows that the most likely one-quarter change is "no change" (with one exception, CP16). For example, p_{11} in the transition matrix in Exhibit 1 is 0.871, far higher than 0.114 for p_{12} and 0.012 for p_{13}. Suppose that CP1's status as the trough of recession with the lowest of the city's occupancy levels taxes financial endurance and brings impatient hope for change. Plausible questions to pose to a real estate cycle analyst are, "How much longer are we going to just stay in this same cycle point? When is the first time that we will move out of these conditions?"

The answer is that staying time is a random variable. Given that we are in CP1, the stay cannot be zero

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or negative, but we might move away in one quarter. We can calculate this probability that the escape is only one step into the future as one minus the probability of staying, $0.129 = (1 - p_{11})$. Then, for an escape exactly two quarters ahead, there is only one scenario: first stay, then escape. This compound event has a probability of $0.112 = (p_{11})(1 - p_{11})$. For longer stays, prob[stay 3] = $(p_{11})^2(1 - p_{11})$, prob[stay 4] = $(p_{11})^3(1 - p_{11})$, ... , prob[stay k] = $(p_{11})^{k-1}(1 - p_{11})$.

Strictly positive random count variables with this type of probability distribution form a class called geometric distributions. The distribution has only one parameter: the probability p_{ii} in the case of real estate cycles. These probabilities are along the diagonal of the P matrix in Exhibit 1. The mean and variance of the random variable differ at every cycle point, but are $\mu_i = 1/(1 - p_{ii})$ and $\sigma_i^2 = p_{ii}/((1 - p_{ii})^2)$. Staying time means and standard deviations are in Exhibit 3.

Prior research focused on the pattern of mean staying times over cycle points and across property types (Evans and Mueller, 2013; and forthcoming). This report's revised estimates in Exhibit 3 use a larger data history for industrial property than was available for the prior estimates. A main generality of the earlier research is replicated, that the real estate cycle seems to speed up and slow down at different cycle points. The longest staying time is at the trough of the cycles, but then the pace increases in recovery as staying times shorten. Having the defining characteristic of seeing long-term average occupancy rates, CP6 has a longer staying time, but then the early expansionary cycle points have low mean staying times—speeding up. CP 9 has the highest mean staying time in the most profitable stages of the Expansionary Phase. After a relatively long stay at the cycle point with supply and demand growing at the same pace, mean staying times are at their shortest in hyper-supply and recession.

This realistic characterization of the real estate cycle "momentum" is not the product of the stochastic element used here. The model used here uses only one past period and a set of probabilities of one-quarter changes. For a Markov chain to show momentum by itself, two or three past periods must be included in the stochastic model. The realism of the pattern seen here for staying time must come from the realism of the underlying real estate cycle model.

The Stationary Vector and First Recurrence Time

For the class of Markov chain models specified for industrial real estate cycles, the matrix power P^n converges to a "characteristic limiting matrix," W, as n goes to infinity. Each row of W has the same set of elements, in the same order, and summing to one. This repeated row is labeled w, and is the particular Markov process's "stationary vector," also called its "fixed row vector": $w = (w_1\ w_2\ w_3\ w_4\ w_5\ w_6\ w_7\ w_8\ w_9\ w_{10}\ w_{11}\ w_{12}\ w_{13}\ w_{14}\ w_{15}\ w_{16})$.

Theorems show that w is a probability vector in the same category as any p^k. In fact, a theorem proves that $wP = w$ if the process is "regular." Thus, the stationary vector, w, has an interpretation as a long-term, equilibrium probabilistic forecast. For the Markov process that generates P, if a forecast k steps ahead ever reaches a step where $p^k = w$ describes the probabilities across points, then the next period will also have that same distribution of probabilities over its states; $p^kP = wP = w$. The process will have reached "stationarity," or reached a "mixing point."

If the underlying stochastic process is a regular Markov chain process, then W may be calculated as the solution to a set of simultaneous equations, or it may be approximated by raising P to a very high power. The values reported in Exhibit 4 were generated by raising P to a very high power in a spreadsheet application. That generates the same results as the solution generated by specialized mathematical software. The elements of each row converge to the same values, expressed to ten decimal places. Since this does not happen by coincidence for matrices that do not describe regular Markov chains, it adds to our confidence that modeling this real estate cycle as a Markov chain has validity.

W and w play important roles in calculating the time that it takes to move through alternative points of

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Exhibit 3 | Industrial Real Estate Cycle Staying Times in Quarters

Cycle Points	p_{ii}	Mean	Std. Dev.	Median	Mean/Median	Mean − Median
1	0.871	7.78	7.3	5.04	1.54	2.74
2	0.739	3.84	3.3	2.30	1.67	1.54
3	0.739	3.84	3.3	2.30	1.67	1.54
4	0.678	3.11	2.6	1.79	1.74	1.32
5	0.597	2.48	1.9	1.34	1.85	1.14
6	0.675	3.08	2.5	1.77	1.74	1.31
7	0.573	2.34	1.8	1.24	1.88	1.10
8	0.654	2.89	2.3	1.63	1.77	1.26
9	0.724	3.63	3.1	2.15	1.69	1.48
10	0.660	2.94	2.4	1.67	1.76	1.27
11	0.687	3.20	2.6	1.85	1.73	1.35
12	0.444	1.80	1.2	0.85	2.11	0.94
13	0.382	1.62	1.0	0.72	2.25	0.90
14	0.397	1.66	1.0	0.75	2.21	0.91
15	0.4807	1.92	1.3	0.94	2.04	0.98
16	0.293	1.41	0.8	0.56	2.51	0.85

Notes: The count number of quarters required for the first move out of a given cycle point is a strictly positive, integer, random variable. The mean and variance can be a fraction because they are mathematical expectations. Here, the median is a fraction so as to indicate that there is no integer that has exactly 50% of the distribution higher and 50% lower. For example, CP1 has a 0.4974 cumulative probability at a stay of 5 quarters, and 0.562 at 6 quarters. The 5.04 median indicated reflects an interpolation between 5 and 6. The 0.56 median for CP16 reflects that 0.707 is the cumulative probability at a one quarter stay. Medians in later exhibits appear as integers.

Exhibit 4 | Fixed Vector and Mean First Recurrence Times, Compared to the Median and Other Quantiles: Industrial Cycles

Cycle Points	w, Fixed Vector	Mean	25%	50%[a]	75%	90%	99%	% < Mean	Mean/Median
1			1	1	1	8	70	89%	4.65
2	0.121	8.26	1	1	3	30	82	80%	8.26
3	0.093	10.70	1	1	3	41	103	78%	10.7
4	0.068	14.71	1	1	18	53	121	74%	14.7
5	0.037	27.04	1	1	41	89	208	69%	27.0
6	0.050	19.84	1	1	27	71	167	73%	19.8
7	0.037	26.71	1	1	43	86	190	67%	26.7
8	0.045	22.24	1	1	29	79	193	74%	22.2
9	0.063	15.88	1	1	3	58	189	82%	15.9
10	0.048	20.67	1	1	7	74	235	80%	20.7
11	0.064	15.71	1	1	3	55	203	84%	15.7
12	0.046	21.67	1	2	30	72	170	72%	10.8
13	0.027	36.91	1	11	58	105	224	63%	3.36
14	0.014	69.37	1	39	104	190	407	64%	1.78
15	0.039	25.61	1	7	41	73	151	63%	3.66
16	0.031	32.21	1	23	50	78	150	60%	1.40

[a] Median.

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interest for a Markov chain. The means of *First Recurrence Time* differ by cycle point and across property types. However, given cycle point i, the mean First Recurrence Time is simply $r_i = 1/w_i$, where w_i is the appropriate element of the stationary vector. Exhibit 4 reports w for industrial property cycles, along with mean first recurrence times.

First Passage Time

First Passage Time is a related random variable, a count of the number of quarters that the Markov chain process starting at cycle point i takes to first reach a different cycle point, j. This application of Markov chain models would be of interest to real estate investors and developers who want to match long-term commitments (that must be made now) to their anticipations of when the cycle conditions will evolve to make them successful. On the other hand, decision makers may want information on how long it may be before a ruinous real estate cycle point will emerge, one perhaps coming before their project has built enough internal strength to persist in harsh cycle conditions.

By convention, when $i = j$, the count is treated as being zero with certainty, given that the process is already in cycle point i. This makes mean *First Passage Time*, m_{ii}, zero when $i = j$. Mean recurrence times and mean staying times are used to fully describe intuitive random count variables associated with $i = j$.

For $i \neq j$, Kemeny and Snell (1960) solved a set of simultaneous equations to derive a classic calculation for mean *First Passage Time*: $m_{ij} = (1/w_j)(z_{jj} - z_{ij})$, where w_i comes from the fixed row vector for the Markov chain process and z_{jj} and z_{ij} are elements of the particular Markov chain's fundamental matrix: $Z = (I - P + W)^{-1}$, where I is a square identity matrix and P is the Markov chain's transition matrix, while W is the process's limiting matrix. Calculating Z requires use of a spreadsheet's matrix inverse function, or the use of special mathematical software. This calculation was not successful in generating plausible estimates, a result that is common in the Markov chain literature and still part of active research. Hunter (2005) suggests an alternative method for finding mean *First Passage Time*, using the Moore-Penrose generalized inverse. While spreadsheet software does not provide this function, such software as Mathematica provides it with a Markov chain set of applications.

The calculation procedure simultaneously gives mean first recurrence and mean first passage. Mean first recurrence calculations are also identical to the values shown in Exhibit 4, but Exhibit 5a shows rounded values for this mean for the elements such that $i = j$.

The mean *First Passage Time* values reported in Exhibit 5a from a generalized inverse calculation seem much more plausible, while earlier calculations did not. Some earlier calculated values were negative as a mean of a random variable that cannot be negative; others seemed too large. However, seasoned real estate cycle observers may believe that, in their experience, the mean first passage times in Exhibit 5a are still too high.

Means of Count Variables

As a caveat, count variables such as *Staying Time*, *First Recurrence Time*, and *First Passage Time* are random variables. Variability around their means can be expected. Often in Exhibit 3, the standard deviations of the *Staying Time* variable are almost as large as the means. Also, as with many random variables that are counts, they are limited to strictly positive values. The distributions are not symmetric, having a positive skew. A positive skew allows the incidence of low probability cases that have very large count values to cause the mean to be pulled "up and out of the middle" of the skewed distribution.

Thus, a practitioner with judgment and experience with the real estate cycle may think that some means are too high. The means of these count variables are usually higher than more than half of the values that real estate cycle analysts have experienced. They also are higher than the most frequently seen value for the count. Simply put, the mean is higher than the median and the mode in

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Exhibit 5a | Mean First Passage Times (from row i, to column j) and Mean First Recurrence Times (for $i = j$)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	5	11	22	29	52	46	53	60	71	88	85	66	75	118	49	46
2	28	8	15	22	45	39	46	53	64	80	77	59	68	111	42	38
3	33	34	11	13	39	33	40	47	58	74	71	53	62	104	34	37
4	32	35	41	15	32	27	34	41	52	68	65	47	56	98	32	35
5	33	36	45	47	27	24	25	32	43	60	57	39	48	90	35	36
6	34	37	45	51	67	20	21	27	38	54	51	33	42	85	37	37
7	32	35	44	51	74	63	27	21	32	48	45	27	34	77	37	36
8	33	36	44	52	75	68	63	22	19	39	36	18	34	79	37	37
9	31	34	42	50	73	66	66	63	16	36	35	16	32	77	35	35
10	29	32	41	48	71	65	69	72	55	21	21	13	30	75	33	33
11	26	29	38	45	68	62	67	72	66	62	16	9	28	72	30	30
12	22	25	33	40	64	57	63	69	69	70	51	22	24	68	26	25
13	18	21	29	36	60	53	60	66	69	77	68	47	37	69	22	22
14	13	14	23	30	54	48	55	62	72	88	84	64	73	69	23	18
15	8	13	24	31	55	48	55	62	73	89	85	68	77	118	26	10
16	4	12	23	30	53	47	54	61	72	88	85	67	76	119	48	32

this kind of random, count variable. *First Recurrence Time* and *First Passage Time* are complex random variables that do not fit into a "named" type of random variable. However, the mean *Staying Time* is a case that is easy to explain.

As described above, *Staying Time* has a geometric distribution. Each cycle point has its own parameter for the distribution, p_{ii}, the probability of no change from an initial quarter to the next. Zero and negative values of the count are not possible. Since a stay of k quarters has a probability of $(p_{ii})^{k-1}(1 - p_{ii})$, for $k = 1, 2, 3, \ldots$, the mode for a geometric distribution is always 1. This is because $k = 1$ maximizes the geometric probability for any $p_{ii} < 1$. The mode is the most frequent observation over a very long sample period. Also, individual shorter staying times are always more likely than longer ones since the probabilities fall geometrically.

While the mean was given above as $\mu_i = 1/(1 - p_{ii})$, the median of a geometric distribution is $M_i = ln(1/2)/ln(p_{ii})$, using natural logarithms. Exhibit 3 shows medians for each cycle point. All medians are lower than corresponding mean staying times. The degree of difference between means and medians differ by cycle point, reflecting p_{ii}, the probability of no change. CP16 has the lowest p_{ii}, the lowest mean staying time, and the lowest difference between mean and median, 0.85 quarters. CP1 has the highest p_{ii}, the highest mean staying time, and the highest difference between mean and median, 2.74 quarters. On the other hand, proportional values show the opposite pattern; CP1's mean is 1.54 times the median, while this ratio for μ_{16}/M_{16} is 2.51 for CP16.

Thus, experienced real estate cycle observers may object to a mean of a count variable reported here as being inconsistent with their experience. The mean is larger than the most likely count, while the mean is higher than more than half of the counts that they may have observed. Unfortunately, mean is the measure most likely to occur in the Markov chain literature. Mean and median are highly correlated, which does allow us to qualitatively generalize about the count variable differing across markets or cycle conditions by referring to means.

First Recurrence Time is a random variable, the time that it takes to first return to a specified cycle point, given that a market is initially in that same cycle point. Negative times are not possible. The mode is often one quarter for industrial real estate cycles (CP16 is an exception), consistent with discussions of mean staying times. Two quarters is also a high

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Exhibit 5b | Median First Passage Times (from row *i*, to column *j*) and Median First Recurrence Times (for $i = j$)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	6	14	20	35	33	39	45	54	66	65	54	60	89	40	38
2	21	1	6	12	27	24	31	37	46	59	57	46	53	82	33	30
3	27	29	1	4	18	16	23	29	39	52	50	39	46	75	25	29
4	26	30	35	1	7	8	15	21	31	44	43	32	39	68	23	27
5	27	32	39	41	1	4	7	11	19	33	32	24	30	59	26	28
6	27	32	39	44	55	1	4	7	13	25	25	20	25	52	27	28
7	26	30	37	43	59	53	1	3	8	17	19	15	19	42	26	27
8	26	30	38	44	60	57	54	1	3	11	13	11	18	44	26	27
9	24	28	35	42	58	55	55	52	1	8	11	9	16	42	24	25
10	22	26	33	40	56	53	57	59	40	1	4	7	13	40	22	23
11	18	23	30	37	53	50	55	58	51	42	1	3	10	36	18	19
12	13	18	26	32	48	45	50	55	53	50	31	2	4	32	13	14
13	8	14	21	27	43	41	46	51	53	58	51	38	11	35	7	8
14	5	9	16	22	37	35	41	47	55	66	65	53	59	39	5	4
15	3	9	16	22	37	35	41	47	56	68	66	56	62	89	7	2
16	1	7	15	21	36	34	40	46	55	67	66	55	61	90	40	23

probability observation, with many possible patterns of moving away from an initial cycle point for one quarter, and then immediately reversing that change to have first recurrence two quarters in the future. However, the random count variable has several notable, very large possible values. If the recurrence does not happen until real estate market conditions loop through all the other cycle points, then the count will be very large. Thus, first recurrence time is a random variable with a distribution showing positive skewness.

Exhibit 4 shows that CP1 has the shortest mean recurrence time. With $p_{11} = 0.871$, a recurrence time of 1 has that same probability, making one the mode at CP1. Specialized software can also generate other quantiles (percentiles), such as the first quartile, the second (the median), and the third quartile (all being reported as 1 in Exhibit 4 for CP1), as well as 8 reported as the quantile such that 90% of the distribution is lower than the 8 shown for CP1, and 70 reported as the number of quarters such that 99% are lower for CP1. Even CP1 has a clearly positively skewed distribution, with most of the distribution being at a one quarter first recurrence time, but with the mean being pulled far from the center of the distribution by non-zero probability occurrences of high first recurrence times.

CP14 has the largest mean first recurrence time in Exhibit 4. It has a relatively low probability of no change quarter-to-quarter, $p_{14,14} = 0.397$, and it also has low probabilities of "going backward" in the real estate cycle model. There are just two observations of this in our data history. Specialized software reports that 64% of the distribution falls below the mean first recurrence time, shown in Exhibit 4. The mean first recurrence time is 1.68 times the median, a clear indication of positive skewness. CP13 has a lower $p_{13,13} = 0.382$, but a lower mean first recurrence than seen for CP14. CP13 has a much greater probability of going backwards than seen for CP14. This raises the probabilities of short recurrence times as the real estate cycle moves backward to CP12 or 11, and then returns to CP13 for a relatively short recurrence time.

Exhibit 1 shows that some cycle points are likely to be skipped altogether as market conditions evolve. For example, CP14 has a 0.182 probability of following CP13 after one quarter ($p_{13,14} = 0.182$), but there is a 0.173 probability of a move from CP13 directly to CP15 ($p_{13,15} = 0.173$). This is evidence supporting the earlier specification decision that the Markov chain model of the industrial real estate cycle is not a cyclical Markov chain. It leads to the insight that leaving CP14 means that there may not

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Exhibit 6 | First Passage Times: Selected Cycle Points

Cycle Points	25%	50%[a]	Mean	75%	90%	99%	% < Mean	Mean/Median
1 → 6	18	33	46.0	60	99	195	65%	1.39
1 → 8	26	45	60.3	79	124	238	64%	1.34
1 → 11	39	65	84.5	109	168	316	63%	1.30
2 → 6	12	24	38.9	52	91	188	65%	1.62
2 → 8	19	37	53.2	71	116	230	64%	1.44
2 →11	32	57	77.4	102	161	308	63%	1.36
3 → 6	7	16	32.6	45	83	180	67%	2.04
3 → 8	14	29	46.9	64	109	223	64%	1.62
3 →11	26	50	71.1	95	154	301	64%	1.42
4 → 8	9	21	40.7	56	102	216	66%	1.94
4 → 11	21	43	65.0	88	147	295	64%	1.51
5 → 8	6	11	32.3	44	90	204	69%	2.94
5 → 11	16	32	56.9	78	136	284	65%	1.78
6 → 8	4	7	26.7	34	80	194	72%	3.82
6 → 11	13	25	51.4	70	129	276	67%	2.06
7 → 11	10	19	44.9	61	119	267	68%	2.36
8 → 11	7	13	36.4	44	103	251	72%	2.80
9 → 11	5	11	34.5	42	101	249	72%	3.14
10 → 11	2	4	21.0	12	68	216	80%	5.26
11 → 14	10	36	72.1	103	189	406	65%	2.00
11 → 1	11	18	26.1	33	59	111	66%	1.45
12 → 14	5	32	68.2	99	185	402	65%	2.13
12 → 1	7	13	21.6	28	51	106	67%	1.66
13 → 14	2	35	68.5	101	187	404	65%	1.96
13 → 1	5	8	17.6	22	46	101	70%	2.20
14 → 1	3	5	13.2	12	39	95	77%	2.64
15 → 1	2	3	7.8	5	17	77	84%	2.59
16 → 1	1	1	3.9	2	4	61	89%	3.86

[a] Median.

be any recurrence of CP14 when the market conditions first evolve through the rest of whole cycle. A high likelihood of "skipped" cycle points generates elevated skewness, making mean first recurrence time a poor measure of how long industrial real estate cycles are in general.

To generalize, mean first recurrence is higher if a cycle point is very likely to see moves forward in the cycle model relative to the probabilities of stays or backward moves. This characteristic would mean that the probabilities would be higher for the very high first recurrence time counts that would come from going through the entire cycle before returning to the initial cycle point. Mean first recurrence is also higher if a given cycle point is more likely to be skipped in market evolutions because multiple loops through the set of cycle points may occur before a first recurrence.

Medians are in Exhibit 5b for all the 240 possible first passage times, while the median first recurrence times appear as diagonal elements of that matrix. Exhibit 6 shows mean first passage times in comparison to the same set of quantiles for selected cycle point pairs. Positive skewness in this distribution is clear, with 63%–89% of the selected distributions being lower than the mean and with the mean

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being 1.30–5.26 times the median. The same set of generalities apply to first passage times as described in the prior paragraph for first recurrence times.

We can generalize that medians are a much superior descriptive statistic than the mean for all these intuitively appealing count variables. We can also generalize that investors who try to time the real estate market face unique, complex distributions that describe the risks that they take in trying to anticipate the time for various cycle conditions to evolve. Cycle speculators may not be un-nerved by high standard deviations and the complex risks posed by trying to time the market precisely, but nonspeculators may see the wide distribution of alternative first passage and recurrence times as validation of their alternative investment philosophies.

Conclusion

This research adds long available, standard tools of Markov chain theory—first recurrence time and first passage time—to the real estate cycle literature. Insights about the complexity of those variables add to the understanding of staying time, which had been applied in earlier real estate cycle research. Now it is clear that all three random variables have probability distributions that are positively skewed. Thus, the medians reported here are more descriptive than the means. The distributions of first recurrence time and first passage time are complex enough that their means should not be used to characterize the overall length of real estate cycles for industrial property.

Perhaps the greatest value of adding a Markov chain element to the real estate cycle model is the generation of forecasts that are not the point forecasts of econometric models, but are probabilistic in format. The revised format for such a cycle position forecast is an explicit numerical probability of it occurring in that future quarter.

The Markov chain model may generate these forecasts for an individual future period (perhaps describing cycle risk conditions when an industrial property would be offered for sale), or for spans of future periods (for an application to anticipating cash flows from operating over four quarter spans). Discounted cash flow analysis becomes more important in this setting. Instead of making market trend assumptions that are trending toward average conditions or staying constant, the analyst would need to recognize that alternative real estate cycle conditions have large enough probabilities that each one should be treated with its own discounted cash flow calculations. In fact, numerical examples shown here show that no change over even medium length investment horizons do not have high, dominating probabilities. An assumed trend toward average conditions may be hard for a reader to dispute, but these cycle points do not have high probabilities in the five year numerical examples here.

These forecasts are direct calculations that use matrix functions available in most spreadsheet programs. All three count variables' means can be calculated with standard spread sheet matrix functions. Only mean first passage calculations from standard spreadsheet software are found unusable. Specialized software does offer more plausible means for first passage times across cycle points, but that specialized software's added strength is to more clearly report medians and the probabilities of individual values for first passage and first recurrence times.

Just as appraisers' principle of contribution of elements of comparison makes hedonic pricing models realistic, Mueller's (1995) model of cycle points is the base for the realism in this application. A qualified real estate cycle analyst may have built a data set of supply and demand drivers. That analyst could explain why a city market changed cycle conditions in the past or why a future change could be predicted.

A Markov chain model may suitably describe uncertainty for many real estate participants who do not have exceptional econometric skills and data. In the numerical examples shown here, we assume that there is certainty about current, initial real estate cycle conditions. Also required are the transition probabilities. A sample of 3,780 quarter-to-

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quarter changes for industrial markets in more than fifty cities supplies the transition probability estimates applied here. Thus, a Markov chain model would describe the uncertainty of a market participant with only these two types of information. The Markov chain element of the model could be more sophisticated, if the participant has the ability to improve on the Exhibit 1 empirical transition probability estimates with common sense theory, or other statistical models for generating transition probabilities.

A synthesis of real estate cycle theory and probability theory might yield benefits and lower analytical costs. The application of count variables that have intuitive appeal to real estate investors is just another example of how real estate analysts can add stochastic elements to a valid real estate model, and get off-the-shelf statistical methods that enhance our understanding of real estate markets.

Appendix
Updating Exhibit 1 with New Cycle Data

Mueller's *Cycle Monitor* also allows later researchers to update the data by requesting copies of the *Cycle Monitor*, and building an update tally matrix for each added quarter. Starting at some selected cycle point, for example CP11, count the number of city markets that are named in the industrial market analysis as being at CP11, and where the city name has no positive or negative integer after it. That lack of an integer indicates that it is one of the number of cities with industrial markets that were in CP11 in the prior quarter and also in the same cycle point in the new quarter. Record the count of these "no change" cities in row 11, column 11 in an "update tally matrix" for the quarter, a matrix with 16 rows and 16 columns. If a city market is labelled as being in CP11, but a +1 appears after its name, then the *Cycle Monitor* is reporting that the city had been in CP10, but now is in CP11. A +2 indicates a transition from CP9 to CP11. A −1 would indicate a city to add to the count of cities that had transitioned from CP12 to CP11. Record these counts in the update tally matrix for the quarter. A given row is associated with the cycle point from which city markets were in the prior quarter, while a given column applies to the cycle point that cities are located at for the quarter of the *Cycle Monitor*'s report. The counts just described for cities listed under CP11 would fill the 11^{th} column for the quarter's update tally matrix. Most entries are zeros. Other columns would be populated with tallies of how many markets had transitioned to each of the other cycle points. Finally, the update tally matrix would be added to the historical tally matrix in Exhibit 1, our history of 3,780 quarter-to-quarter changes in cycle conditions for industrial markets.

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