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MICHEL BARONI
FABRICE BARTHELEMY
MAHDI MOKRANE

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Michel Baroni *

Fabrice Barthélémy **

Mahdi Mokrane ***

* ESSEC Business School, Avenue Bernard Hirsch – B.P. 105, 95021, Cergy-Pontoise Cedex, France.
Tel: (33) 1 34 43 30 92. Mail: baroni@essec.fr.

** THEMA, University of Cergy-Pontoise, 33, Bd du Port, 95011, Cergy-Pontoise Cedex, France.
Tel : (33) 1 34 25 62 53. Mail : fabrice.barthelemy@u-cergy.fr.

*** IXIS-AEW Europe, 12/20 rue Fernand Braudel 75013 Paris.
Tel (33) 1 58 55 32 40. Mail : mahdi.mokrane@ixisaew.com.

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Abstract

This paper considers the use of simulated cash flows to value assets in real estate investment. We motivate the use of Monte Carlo simulation methods for the measurement of complex cash generating assets such as real estate assets return distribution. Important simulation inputs, such as the physical real estate price volatility estimator, are provided by results on real estate indices for Paris derived in an article by Baroni, Barthélémy and Mokrane (2005). Based on a residential real estate portfolio example, simulated cash flows (i) provide more robust valuations than traditional DCF valuations, (ii) permit the user to estimate the portfolio's price distribution for any time horizon, and (iii) permit easy Values-at-Risk (VaR) computations.

Keywords: DCF, Monte-Carlo Simulations, Real Estate Indices, Real Estate Valuations

Résumé

Ce document de travail présente la façon d'utiliser des cash-flows simulés pour évaluer des actifs immobiliers réels. Nous montrons que l'on peut utiliser des méthodes de simulation de Monte-Carlo pour déterminer la valeur d'actifs immobiliers dont la génération de cash-flows est complexe. Les données principales de la simulation telles que l'estimation de la volatilité du prix des actifs réels, proviennent des résultats établis lors de l'élaboration d'un indice immobilier pour Paris, dans un article de Baroni, Barthélémy et Mokrane (2005). Établis à partir d'un exemple de portefeuille immobilier, les cash-flows simulés permettent (i) d'élaborer des évaluations plus robustes que les évaluations traditionnelles résultant de méthodes de DCF, (ii) d'estimer la distribution du prix du portefeuille sur des horizons de détention variés (iii) et de calculer aisément des Values-at-Risk (VaR).

Mots-clés : DCF, Évaluations immobilières, Indices immobiliers, Simulations de Monte-Carlo

JEL Classification code: C15, G12

Introduction

In this paper we argue that appropriate modelling of real estate assets coupled with Monte Carlo simulations methods may be of great use in real estate finance (investment management, portfolio management and risk management).

Monte Carlo simulation techniques are often used in finance whenever the value of an asset or a derivatives product is difficult or impossible to compute using analytical solutions. Examples of assets whose values are difficult to compute directly but can be estimated using simulations include exotic options such as barrier options, options on the average price, look-back options or other complex cash flow generating assets such as a power plant.

We consider a real estate asset to be a complex cash flow generating asset, since, as we will see in the next sections, its cash flows are subject to several sources of risks, some of which may even be correlated. Thus the cash flow generating stochastic process is complex to estimate using closed-form formulas, but, as will be shown, can be relatively simply estimated using Monte Carlo simulations.

Once the cash flow generating process is estimated, the value of a real estate asset as well as almost any type of derivatives of this asset, can be valued.

This approach has been followed by Quigg (1993) who realized one empirical study that compares NPV with real options methodology in the context of real estate investment. The author studies 2,700 land transactions in Seattle and finds empirical support for a model that incorporates the option to wait to develop land. The owner of the undeveloped property has a perpetual option to construct an optimal-sized building at an optimal time. Quigg builds an option model with two sources of uncertainty—the development cost (exercise price) and the price of the building (the underlying asset). Her test procedure was to collect 2,700 land transactions in Seattle between 1976 and 1979 and to break the sample into five categories (commercial, business, industrial, low-density residential, and high-density residential)¹.

Quigg's results support the option-pricing approach. The option model prices were, on average, 6 percent above the intrinsic value suggested by the regressions. In a "horse race" where actual transaction prices are regressed against either the option value or the regression value, the option model *r*-squared was higher in 9 of 15 cases, and the slope coefficients in the option regressions were closer to one in seven out of 15 cases. Furthermore, when the option premium was added to the multiple regression, it was a significant variable in 14 out of 15 cases.

More recently, Li (2000) shows the advantages of using simulations methods for land appraisal. Then, Glicksman and Greden (2005) propose a model based on real options techniques for valuing managerial flexibility of space.

Traditionally, the methods used to appraise buildings or real estate portfolios are based on Cash Flows analysis (Discounted Cash Flows). The cash flows usually include inflows (rents), outflows (various expenses) and a peculiar inflow at maturity represented by a terminal value (resale value).

¹ Quigg used regressions to estimate property prices as a function of building and lot sizes, building height and age, and dummy variables for location and season. Third, the standard errors of the regressions were used to estimate the variances needed for the option model, namely the variance of developed property values and of development costs. The final step was to calculate option-based prices, assuming that the building would be built (i.e., that the option would be exercised) when the ratio of its price to the development cost was greater than (one plus) the market rate of interest.

In this paper, we use Monte Carlo simulation techniques to estimate the value of a real estate portfolio and we compare our results to those obtained by a Discounted Cash Flow method. We insist on the specific weight of the terminal value in the portfolio valuation and we try to determine the most robust approach.

In Section 1, we present the variables which are influent in the real estate portfolio valuation. We then expose in Section 2 the available methods to evaluate the free cash flows (DCF and Monte Carlo simulations). In the DCF method, we underline how the results are dependent on the terminal value choice. After illustrating our Monte Carlo methodology through an example (Section 3), we analyse the sensitivity of the two methods on assumptions (Section 4). We finally show how the Monte Carlo approach can be used to determine at-maturity distributions for the portfolio and to compute Values-at-Risk (Section 5).

1 Portfolio Valuation

The two portfolio valuation methods we compare in this article are based on free cash flows actualisation. In these two approaches we deal with the same issues of elaborating free cash flows and choosing the discount rate for actualisation. The main difference comes from the way of computing the cash flows.

In fact, the risk associated to the price of an asset is only one of several sources of risk affecting portfolio's cash flows. We propose to model portfolio's cash flows using five main sources of risk:

- the rental rate risk which comes from the market (more precisely from the evolution of rent prices represented by a rent price index)
- the rental occupation rate which is more specific and also comes from the market (supply and demand confrontation)²
- the current expenses evolution
- the capital expenditures evolution
- the price risk which is a market risk (more precisely represented by a capital growth index)

The conjunction of these various sources of risk leads to a risk structure for the portfolio's cash flows. The first two sources of risk concern inflows, the following two concern outflows and the last one by its weight in valuation, the asset terminal value (particular inflow which occurs only once at the resale of the portfolio).

1.1 Cash Inflows (rents expectation)

The cash flows generated by a real estate investment generally take the form of rents payments. In estimating rents for future years, several factors have to be considered, including:

- past trend in rents,
- demand and supply conditions for space provided by the properties,
- general economic conditions as the impact of rent control laws on rent evolution.

All those elements are taken into account by the rents index. Let's define $Rent_t^*$ the expected *potential* rent payments at time t .³

² We can notice that this risk is portfolio dependent but it can be considered systematic for a large given portfolio.

³ This corresponds to the expected payment if the asset is occupied.

In residential buildings, all space may not be rented at a particular time. Thus, the vacancy rate (i.e., the percentage of the space that is not rented out at any point in time) has to be projected in conjunction with market rents. Even in tight markets, there will be periods of time when space cannot be rented out, leading to a positive vacancy rate. Let's denote η_t the occupancy rate at time t . We can define the expected rents ($Rent_t$) payments by combining these two risks: the rent level and the occupancy rate:

$$Rent_t = \eta_t \times Rent_t^* \quad (1)$$

1.2 Cash Outflows (expenses expectation)

As for inflows, we can separate two kinds of expenses:

- the current expenses: linked to regular expenses as insurance, repairs and maintenance. It is empirically well known that these expenses are linked to the rent payments. Let us define Exp_t^* the expected *potential* current expenses at time t .⁴ As for the rents, we can combine this risk (incertitude on the level of expenses) with the probability to have such expenses (ν_t).⁵ Then the expected current expenses at time t can be expressed as:

$$Exp_t = \nu_t \times Exp_t^* \quad (2)$$

- the capital expenses: they correspond to very occasionally high expenses related to 'works' as for instance, roof reparation, ... Let us define Wk_t the expected potential 'works' (capital expenditure) at time t and κ_t the probability of such works. These expected capital expenses at time t can be formulated as:

$$Wk_t = \kappa_t \times Wk_t^* \quad (3)$$

1.3 Free cash flows

If we set that we are at period 0, we are interesting on evaluating all the (future) flows from period $t \in]0, T]$. For each future period t except the last period T , the free cash flows are:

$$FCF_t = (1 - \tau)(Rent_t - Exp_t - Wk_t) + \tau Dep_t, t \in]0, T[\quad (4)$$

where Dep_t is the asset depreciation and τ is the tax rate. By using the occupancy rate, this becomes:

$$FCF_t = (1 - \tau)(\eta_t \times Rent_t^* - Exp_t - Wk_t) + \tau Dep_t, t \in]0, T[\quad (5)$$

To determine the free cash flow at time T , the end of the holding period, we need P_T , the expected asset terminal value. Then,

⁴ Conditionally to the fact that there are expenses.

⁵ We can notice that the probability of such expenses is quite high and even equals to 1 in presence of maintenance for instance.

⁶ We assume the capital expenses are not considered as an asset and may not be depreciated.

$$FCF_T = (1 - \tau)(\eta_T \times Rent_T^* - Exp_T - Wk_T) + \tau Dep_T + P_T - \tau \times PV \quad (6)$$

where PV represents the plus-value at the end of the investment.

1.4 Discount Rate Choice

The distinction between cost of equity and cost of capital is significant. If the cash flows being discounted on a real estate deal are cash flows to equity, the appropriate discount rate is the cost of equity. In our analysis, the cash flows being discounted are predebt cash flows, that is, cash flows to the firm, and then the appropriate discount rate is the cost of capital.

The cost of capital is the average of the cost of equity and the after-tax cost of debt, weighted by their market value proportions.

We have now the way of discounting the free cash flows in our two different approaches. Let's denote k the weighted average cost of capital (WACC).

2 How can we evaluate free cash flows? DCF or simulations

If we determined in section 1.3 the free cash flows in a conceptual sense, we have not defined the way to evaluate them empirically. What about the rents, the expenditures and moreover, what about the terminal value? Two radically different answers can be made which lead to two different evaluation approaches:

- The classical DCF approach
- The simulation based approach (we propose)

As we are dealing with a portfolio, each free cash flow can be represented by an average value.⁷ For each flow, we can set an a priori value of this average value (DCF) or we can compute its value from a flow simulated distribution (simulation based approach).

2.1 DCF

Different (average) values are set for the components of equations (5) and (6). We first analyse the asset regular flows at each time, and then in a second part, analyse more precisely the way of computing the terminal value.

2.1.1 Rents, expenditures and works

We have to suppose ad hoc values for the asset inflows and outflows and to make assumptions on the evolution of rents, current expenses and capital expenses. We assume (as it is usual in practice) that the rents and expenditures growth are constant. This assumption relies on the fact that without more information the evolution is supposed to be the same over the time.

We specify:

- the average rents growth rate g_{Rent} which has to be combined with the “probability of occupancy”,
- the average occupancy rate $\bar{\eta}$,

⁷ It can be the average value of the market or more precisely to a sub-market from which the portfolio is built. For instance, in the case of a residential portfolio, only the values on the sub real estate market of residential property would be of interest. This can be extended to more detailed sub markets according to the portfolio constitution.

- the current expenses growth rate g_{Exp} , which have to be combined with the “probability of assuming current expenses” for the portfolio, \bar{v} .
- the capital expenses evolution g_{Wk} which have to be combined with the “probability of assuming capital expenses” for the portfolio, \bar{k} .

Hence, for each future period t except the last period T , the free cash flows in the DCF approach are:

$$FCF_t = (1 - \tau) \left(Rent_0 \times \bar{\eta} \times (1 + g_{Rent})^t - Exp_0 \times \bar{v} \times (1 + g_{Exp})^t - Wk_0 \times \bar{k} \times (1 + g_{Wk})^t \right) + \tau Dep_t \quad (7)$$

where $Rent_0$, Exp_0 and Wk_0 are respectively the initial values for the rents, the current expenses and the capital expenses.

2.1.2 Terminal value

In the DCF approach, it is usual to use an infinite growth rate, g_∞ for last cash flow FCF_T of the period]0,T] to calculate the terminal value. We assume that the free cash flows in the terminal year (the last year of the investment horizon) will continue to grow at a constant rate forever after that date. If this assumption is made, the terminal value P_T of the asset is:

$$P_T = \frac{FCF_T (1 + g_\infty)}{k - g_\infty} \quad (8)$$

then

$$P_T = \left[(1 - \tau) \left(Rent_0 \times \bar{\eta} \times (1 + g_{Rent})^T - Exp_0 \times \bar{v} \times (1 + g_{Exp})^T - Wk_0 \times \bar{k} \times (1 + g_{Wk})^T \right) + \tau Dep_T \right] \times \frac{1 + g_\infty}{k - g_\infty} \quad (9)$$

We can notice that other methods can be used to compute the terminal value as the expected inflation rate or the “capitalization rate”.

2.2 Simulating a Portfolio’s Free Cash Flows

The methodology is really different from the one we have just developed. It is based not on a priori values but on estimations using Monte Carlo simulation methods. In a first part we present the aim of our methodology and then the estimation method.

2.2.1 Methodology

In this section, we provide a framework in which we simulate, using Monte Carlo method, the dynamics of a real estate portfolio’s free cash flows (say a residential portfolio). The methodology is based on the modelling of rents, expenditures, and price dynamics.

In this approach, we are modelling each flow as a stochastic process, or in a more simple way, as random variable when this process is not time dependent. The model preserves a large degree of freedom for all the considered variables and their eventual links. This modelling may include experiences from the real estate portfolio managers. For instance, for a particular portfolio the asset manager may have empirical knowledge of the occupancy rate dynamics and may then use it to simulate the rents expectation. Higher is the experience better should be the model used for simulations in the sense that the dynamics represented in the model should be closer from reality.

We are not only modelling expenses and rents, but we even represent the real estate prices dynamics. Hence, our approach is significantly different from the classical DCF approach in the sense that we need not to determine a specific value for the terminal value of the asset. Thus, instead of trying to compute the terminal value using for instance the infinite growth rate method, we start from an initial price P_0 , and assume that this price is subject to annual dynamics governed by equation we specify. Hence in our approach, the Terminal Value is not computed as in the usual DCF approach, but is simply the actual quoted price for the portfolio projected into the future using the housing price dynamics.

To illustrate our methodology, we can specify dynamics for prices and rents. For instance we model returns in the form of geometric Brownian motion processes. Indeed, this return behaviour modelling is today very frequent in finance.

We suppose that prices are governed by the following geometric Brownian motion.

$$\frac{dP_t}{P_t} = \mu_p dt + \sigma_p dW_t \quad (10)$$

This equation assumes that housing returns can be modelled as a simple diffusion process where parameters μ_p and σ_p are the trend and volatility.

We suppose in the same way that rents are governed by the following geometric Brownian motion:

$$dRent_t / Rent_t = \mu_R dt + \sigma_R dW_t^R \quad (11)$$

We model the building's occupancy rate at date t , using a random variable, η_t , whose law can be a Uniform on $[a ; b]$ with $0 \leq a \leq b \leq 1$.⁸

To use such a model (10-11), we need to fix or estimate the values of a and b . Moreover we need estimations of the parameters μ_R , σ_R , μ_p and σ_p , and we have to set an initial price P_0 and an initial rent $Rent_0$. Using Monte Carlo simulations based on both hypotheses and estimation, we are then able to estimate the main parameters for the cash flow's dynamics.

2.2.2 Estimating a Real Estate Asset's Cashflow Structure

This estimation method is based on the results provided by the Law of Large Numbers (LLN): the mean of random variables converges to the expectation. We then take the mean of a great number of random variables realisations. Before doing this estimation, we need to define our random variable on one single sample.

A single path

In order to present the methodology that will be extensively used in the rest of the article, let us consider the equation (10). This equation implies that, if we consider date t belonging to an investment period $[0, T]$, the time t distribution for price P_t is a log-normal distribution⁹:

$$P_t = \exp \left\{ \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\} \text{ with } W_t \longrightarrow N(0, t)$$

The Brownian component W_t is defined by:

⁸ As said before, if more information is available (for instance, due to the asset manager experience) we may specify the occupancy rate at time t in a different way.

⁹ See Hull for a description of asset modelling.

$$W_t - W_{t-1} = \xi_t \quad \text{with } \xi_t \longrightarrow N(0,1)$$

Hence,

$$W_t = \sum_{i=0}^{i=t} \xi_i \quad \text{where } \xi_i \longrightarrow N(0,1)$$

The discretisation of the previous continuous process gives:

$$P_t = P_0 \exp \left\{ \left(\mu - \frac{\sigma^2}{2} \right) \times t + \sigma \times \sum_{i=1}^{i=t} \xi_i \right\} \quad (12)$$

To generate brownian processes we use the Maruyana approach (as our parameters μ et σ do not depend on time, we do not need to specify for instance first or second derivatives by time¹⁰).

For example, let us consider the following process:

$$\frac{dS_t}{S_t} = 0.06 dt + 0.03 dW_t, \quad \text{with } S_0 = 100/9 \quad (13)$$

We construct one path for $T = 5$ years, discretising in 1024 increments for each year. Thus, 5120 increments (N) are represented in *Figure 1* and *Figure 2*. This algorithm is converging to the diffusion process as the number of increments for a given period increases. We then have an estimated path \hat{S}_t for $t = 0, \dots, T$.

To estimate the mean and variance return with one path, we estimate for each t the return as:

$$\hat{R}_t = \frac{\hat{S}_t}{\hat{S}_{t-1}}, \quad t = 1, \dots, T$$

and the logarithm of the return as:

$$\widehat{\ln R}_t = \ln(\hat{R}_t) = \ln \left(\frac{\hat{S}_t}{\hat{S}_{t-1}} \right), \quad t = 1, \dots, T$$

Then we use the standard following formulas to estimate the trend and the volatility of the dynamics based on one path. The expected mean of logs is:

$$\hat{m} = \frac{1}{N} \sum_{t=1}^N \ln \left(\frac{\hat{S}_t}{\hat{S}_{t-1}} \right) \quad (14)$$

The return volatility is:

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{t=1}^N \left[\ln \left(\frac{\hat{S}_t}{\hat{S}_{t-1}} \right) - \hat{m} \right]^2 \quad (15)$$

The Brownian's trend is then estimated as follows:

$$\hat{\mu} = \hat{m} + \frac{1}{2} \hat{\sigma}^2 \quad (16)$$

¹⁰ See Roncalli, 1995

In our previous example we estimate $\hat{\mu} = 0.059691673$ and $\hat{\sigma} = 0.030006072$. For each single path simulated we obtain different estimations of these two parameters.

Monte Carlo estimation

The expectation of the random variables \hat{m} , $\hat{\sigma}^2$ and $\hat{\mu}$ are respectively m , σ^2 and μ the true but higher the variance (of these random variables), smaller the precision of estimate. In order to decrease the variance of these three estimators we simulate others paths, to increase the information concerning the true value for the expectation, thus leading to less chances to be far from this true value (the expected return).

Then we generate p paths, for the price P_t , the j^{th} path being denoted P_t^j .

$$\forall j = 1, \dots, p, \forall t = 1, \dots, T, P_t^j = P_0^j \exp \left\{ \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma \sqrt{t} \times \xi_t \right\}, \xi_t \rightarrow N(0,1) \quad (17)$$

Note that this method can not only estimate moments, but also others characteristics of the random variable of concern, such as the distribution of returns for example.

If we estimate the parameters with p paths, the empirical distribution of the p estimated values is an estimation of the distribution function of the expected return. This is also true for the quantiles¹¹ (see Mittelhammer 1996). For instance, the Monte Carlo log return estimator is¹²

$$\hat{m}_{MC} = \frac{1}{p} \sum_{j=1}^p \hat{m}_j \text{ where } \hat{m}_j \text{ is the estimation of } m \text{ with the } j^{\text{th}} \text{ path.}$$

$$\hat{\sigma}_{MC}^2 = \frac{1}{p} \sum_{j=1}^p \hat{\sigma}_j^2 \text{ where } \hat{\sigma}_j^2 \text{ is the estimation of } \sigma^2 \text{ with the } j^{\text{th}} \text{ path.}$$

$$\text{Then } \hat{\mu}_{MC} = \hat{m}_{MC} + 0.5 \hat{\sigma}_{MC}^2.$$

In our example, we obtain the following estimations.

Number of replications	\hat{m}_{MC}	$\hat{\sigma}_{MC}$	$\hat{\mu}_{MC}$
1	0.05719	0.030462	0.05766
500	0.05864	0.029993	0.05909
5000	0.05929	0.029998	0.05974

¹¹ See Mittelhammer 1996

¹² We can notice that the \hat{m}_j variance is an estimator of the variance of the random variable \hat{m}_{MC} . Similarly, the $\hat{\sigma}_j^2$ variance is an estimator of the variance of the random variable $\hat{\sigma}_{MC}^2$.

3 Valuation example

To be able to compare the two approaches (DCF and simulations) in an example we use the same assumptions for the parameters when it is possible, and moreover, we link the values used in the two methodologies to avoid possible biases. First, we describe the example and set values for exogenous variables. Secondly, we present the parameters estimations of the dynamic model we have presented in last section. Then, we conclude with the results on the portfolio valuation according to the methodology.

3.1 Example description

The following example corresponds to a diversified portfolio of residential real estate located in Paris area. The investor is supposed to be a fund not submitted to taxes ($\tau = 0$).

We define P_0 the price at which the investor can buy the portfolio whose value is set to 100. The potential rent is denoted $Rent_0$, whose value is estimated as $P_0/11$.¹³ To set the current expenses at period 0, denoted Exp_0 , we use an empirical relation between current expenditures and rents:

$$Exp_0 = Rent_0/6.$$

The probability of current expenses is set to one. This means that $\bar{\nu} = 1$ in the DCF approach, and that ν_t is a random variable with a degenerate distribution function whose value equals to 1 with probability 1. The initial values Exp_0 is used for the two approaches as an initial value. There are no capital expenditures during this period ($Wk_t = 0$).

We suppose that the portfolio holding period is 5 years ($T = 5$). The WACC (k) is fixed to 8.40% per year.

The occupancy rate is modelled as a uniform random variable whose range is [0.75 ; 0.95] and consequently its average is 0.85 ($\bar{\eta}$).

We set the ‘mean’ potential rents growth rate g_{Exp} at 3%. Other values, as the ‘mean’ potential current expenses growth rate g_{Rent} , or the parameters of the dynamics for our approach, will be estimated. The estimations are presented in next sub section.

3.2 Prior estimations

We estimate the parameters of the equations (10) and (11) μ_R , σ_R , μ_P and σ_P . The dynamic values for these two processes are annual.

3.2.1 Rents

For the rental rate risk measure we use a Paris rental index (the OLAP index). To use this index we estimate its annual trend and its annual volatility. As $\hat{m}_R = 0.0606$ and $\hat{\sigma}_R = 0.0288$ the dynamics for the rents is estimated as follows

$$\frac{d\widehat{Rent}_t^*}{Rent_t^*} = 0.0611dt + 0.0288dW_t^R \quad (18)$$

¹³ In the DCF approach, the P_0 value is not indispensable, but the linkage between initial rent and initial price enables us to give the same magnitude order for the two approaches and makes sense to their comparison.

We deduce from the previous estimation the ‘mean’ rents growth rate g_{Rent} as $g_{Rent} = \exp(\mu_R) - 1$ where $\mu_R = m_R + 0.5\sigma_R^2$ (m_R and σ_R^2 being respectively the expectation and the variance of the returns in logarithm). The estimated value of g_{Rent} is 6.30%.

For 10 simulated paths, we obtain the *Figure 3*. If we combine the potential rent with the occupancy variable (assumed to change every 3 months) the parameters increase:

$$\hat{\mu}_R = 0.0798 \text{ and } \hat{\sigma}_R = 0.1917 \text{ (see Figure 4).}$$

On the other hand, with an occupancy rate of 100% but with current expenses, the trend of log rent returns is estimated to 0.0680 (because of the growth rent of the expenses which is inferior to 6%). In this case, the volatility is estimated to 0.0350 (see *Figure 5*).

3.2.2 Prices

For the price dynamics estimation, Baroni, Barthélémy and Mokrane (2005) used a weighted repeat sales model to estimate price indices for physical real estate capital. The estimated model of real estate returns is based on observed transaction prices, and was then used to generate a time series representing the price evolution for the period 1973-2001. This time series helps to model returns in the form of geometric Brownian motion processes. In Baroni *et alii* the periodicity is 6 months. As the periodicity is annual in our example, we just annualise the semi-annual mean and volatility.

The results are gathered in the following table:

\hat{m}_p (%)		$\hat{\sigma}_p$ (%) (%)		$\hat{\mu}_p = \hat{m}_p + 0.5\hat{\sigma}_p^2$	
<i>Annualised</i>	<i>Annualised</i>	<i>Annualised</i>	<i>Semi-annual</i>	<i>Annualised</i>	<i>Semi-annual</i>
6.78	3.39	5.38	3.80	6.92	3.46

Table 1: Diffusion Parameters for Physical Real Estate Prices

The above parameters imply the following stochastic behaviour for real estate prices:

$$\frac{dP_t}{P_t} = 0.0692 dt + 0.0538 dW_t^P \quad (19)$$

This equation thus assumes that housing returns can be modelled as a simple diffusion process where parameters μ_p and σ_p are the yearly trend and volatility (cf *Figure 6*).

Our approach is based on the fact that a portfolio’s cash flows are subject to various sources of risk: occupancy rate, rent and price. The last two sources of risk are correlated. We have estimated, from 1973 to 2001, the two indices’ correlation to be $\rho = 0.417$. Our simulations will take this correlation into account:

$$dW_t^P dW_t^R = 0.417 dt \quad (20)$$

We can represent the two Brownian dynamics, price and rents, on the same graphic (see *Figure 7*), but in fact, the dynamic of the portfolio is quite different in the sense that only at time T , the cash flow includes the resale. From $[0, T[$ the only dynamics are the rents and the expenses.

If we consider that we can sell the portfolio between 0 and T the potential price dynamics is illustrated according to different values of T (see *Figure 8* to *Figure 10*).

For the two approaches, the numerical formulation is given in Appendix.

3.3 Portfolio valuation results

First, we calculate (DCF method) or estimate (simulation method) the 5 annual free cash flows except the terminal value (TV). *Table 2* presents these values at time $t = 5$ years and *Table 3* at time $t = 0$ (discounted). They clearly illustrate the LLN principle. The expectation of simulated free cash flows (for 50 000 simulations) corresponds to the mean, (the value we set for the DCF). For instance, even if the occupancy rate is a random variable, when the number of paths increases, we get a mean value of 0.85. Hence, the fundamental difference between these two approaches without taking into account the terminal value is the ability of estimating a standard error of the free cash flows, and moreover, a distribution.

	FCF_1	FCF_2	FCF_3	FCF_4	FCF_5
<i>DCF</i>	6.653525	7.12424	7.626158	8.16129	8.731778
<i>Simulation</i>	6.662412	7.14122	7.625426	8.16756	8.753277
<i>Standard error</i>	(0.60796)	(0.690956)	(0.781186)	(0.889103)	(0.983694)

Table 2: Portfolio free cash flows without terminal value

	FCF_1	FCF_2	FCF_3	FCF_4	FCF_5
<i>DCF</i>	6.137938	6.062894	5.98712	5.910738	5.833864
<i>Simulation</i>	6.146137	6.077341	5.98654	5.915276	5.848227
<i>Standard error</i>	(0.560849)	(0.588019)	(0.613291)	(0.643925)	(0.657224)

Table 3: Portfolio discounted free cash flows without terminal value

Table 4 presents the results for the portfolio valuation including the terminal value. We use the fifth cash flow value to compute the terminal value (TV) in the DCF approach:

$$TV = [8.73 / (8.40\% - 3\%)] = 166.55 \text{ millions}$$

Then, we simulate 50 000 free cash-flows series as described by equations (18-20). These simulations provide 50 000 values of cash flows specified in (18) and 50 000 potential resale prices according to the dynamics specified in (19). The mean corresponds to the estimation of the portfolio terminal value (141.23).

<i>Value</i> \ <i>Method</i>	<i>DCF</i>	<i>Simulation</i>
<i>Value at 5 years (at time T)</i>		
Terminal Value	166.55	141.23
Portfolio Value	211.35	186.01
<i>Value at time 0 (discounted)</i>		
Terminal Value	111.27	94.31
Portfolio Value	141.21	124.28

Table 4: Portfolio value at $T = 0$ and $T = 5$ years

Note that the simulation-based valuation yields a present value 12% below the traditional DCF valuation.

The distinctions in the two methodologies are:

- The way the terminal values are computed.
 - In the DCF approach, the terminal cash flow is assumed to grow at a constant rate g_{∞} for infinity.
 - In the simulation-based approach, we use the price dynamics
- The way we represent rents:
 - In the DCF approach, the growth rate is set without links to prices
 - In the simulation-based approach, rents and prices are assumed to have their own stochastic movements and co-movement (recall their instantaneous correlation $\rho = 0.417$).

But in fact, the difference in the portfolio valuation comes essentially from the terminal value valuation as shown in *Table 3* and *Table 4*.

4 Sensitivity and limitations

4.1 Sensitivity DCF vs Simulations

In the DCF approach, the portfolio's terminal value, and thus its total discounted value, is very closely linked (and sensitive) to the assumption concerning the infinite growth rate for cash flows ($g_{\infty}=3\%$). In order to compare the sensitivity to the terminal value estimation, we analyse the portfolio price variations when we modify:

- The infinite growth rate for the DCF approach,
- The price and rent trend parameters for the simulation-based approach.

As an illustration, consider the following table that provides DCF valuations for five different infinite horizon growth rates:

<i>Infinite horizon CF growth rate (g_{∞})</i>	<i>Portfolio valuation (millions)</i>
1%	109.56
2%	122.91
3%	141.21
4%	167.82
5%	210.01

Table 5: DCF Valuation: Infinite Horizon CF Growth Rate (g_{∞}) Sensitivity

The following table mirrors the preceding table assessing the sensitivity of estimated portfolio values with respect to the price trend parameter:

<i>Price trend (μ_P)</i>	<i>Average Simulated Portfolio Value (millions)</i>
4.92%	114.73
5.92%	119.23
6.92%	124.28
7.92%	128.38
8.92%	134.14

Table 6: Simulation-Based Valuation: Price Trend Sensitivity

As we can see the portfolio's present value is much less sensitive to the price trend in the simulation-based approach than in the DCF approach. We consider this to be significant advantage of the simulation-based valuation approach presented here.

Furthermore, *Table 7* shows the sensitivity to the rent trend is rather low.

<i>Rent trend (μ_{Rent})</i>	<i>Average Simulated Portfolio Value (millions)</i>
4.11%	122.27
5.11%	123.79
6.11%	124.28
7.11%	125.12
8.11%	126.10

Table 7: Simulation-Based Valuation: Rent Trend Sensitivity

Finally, we notice that the volatility of the portfolio dynamics (price and rent) does not affect the mean of the portfolio value. Only the mean estimator precision is concerned. Higher the volatility is, smaller the precision.

4.2 Limitations of DCF Valuation

In the sections above, we have pointed out that even if it is much easier to estimate cash flows for real estate than for some financial investments (for instance, a high-growth stock), the DCF valuation is highly sensitive to the infinite growth rate parameter used to estimate the terminal value. This sensitivity is also high if the terminal value is determined using a capitalization rate.

Moreover, it is often argued that DCF valuation does not reflect market conditions, that the market is strong or weak at the time of valuation. However, there are two arguments that may come in favor of DCF. On one level, the cash flows should reflect the market conditions, since they will be higher (higher rents and lower vacancy rates) and grow faster in strong market conditions. On the other level, any additional value being assigned by the market beyond the cash-flow levels can be considered to be "overvaluation" and should not be built into the appraised value in the first place. From our point of view, the DCF weakness comes overall from the fact that DCF valuation does not capture the distributions features of cash flows, and cannot serve to value contingent contracts on the real estate asset. We believe the simulations approach presented in this paper offers a superior precision for valuation by being less sensitive to input parameters.

5 Use of the simulations based approach

5.1 At-Maturity Price Distribution Estimations

For each element of the portfolio valuation we can estimate a distribution function. We may estimate, using the Gaussian kernel estimator, the distribution for a given horizon T of the diffusion processes' possible values. For this purpose we have divided each unit time interval (one year) into 5120 sub-intervals. The total number of simulations is 50 000.

Figure 11 represents for our previous example the estimated density of the five annual free cash flows (without the TV) for a 5 years period. The distribution of the terminal value at 5 years is presented in *Figure 12*. Then, the portfolio value at maturity has an estimated distribution function shown in *Figure 13*. We can then deduce an estimation of the cumulative distribution function for the portfolio value at time T (see *Figure 14*).

The above simulations enable the production of the following information that may be useful in decision making for the portfolio manager.

Lowest value	134,16
Highest value	273,08
Mean	189.48
Probability of being lower than 160	4,12%
Probability of being higher than 220	5,16%

Table 8: Housing price values in 5 years based on 50 000 simulations

5.2 Implications for Value-at-Risk (VaR) in Real Estate Finance

5.2.1 Presentation

Value-at-Risk (VaR) is a method employing standard statistical techniques for the purpose of estimating a given portfolio's risk (see Jorion, 1996). Formally, VaR measures the maximal loss that may happen during a given time period in normal market conditions and for a given confidence interval. A portfolio's VaR takes the form of a unique number, computed with reference to both a holding period and a confidence level.

Time Horizon

The holding period corresponds to the horizon at which the investor considers potential losses. Naturally, the change in portfolio or asset value depends on the time allowed for the potential change to take place. Note that as stated by Jones (1996), one expects a one day VaR to be lower than a 5 day VaR. However, for much longer periods of time, for which the trend may overcome the volatility of a random price time series, VaR will decrease with time.

The choice of a holding period is a significant aspect of portfolio management and risk management. For the risk measure to be significant one will seek to select a holding period as close as possible to the usual portfolio appraisal value of the investor or institution (a given number of trading days for a derivatives portfolio, from a couple of months to several years for long term portfolios or real estate investments). Ideally, the holding period should correspond to the longest time interval necessary to liquidate the whole portfolio (see Jorion, 1996).

The Confidence Level

The fact that a VaR measure is computed using a confidence level implies that the total loss may be greater than what the VaR measure initially estimated. The frequency with which this loss will exceed the VaR depends on the level of confidence chosen.

Depending on the assumptions concerning the portfolio's risk, the confidence level will take a different signification. Given a normal distribution of market risk (possible at-maturity values), the confidence levels will have a simple scalar relation. However, if the distribution is not normal, the choice of a wrong confidence level may hide important risk elements.

What's more, the choice of a horizon length also depends on how the investor plans to use the VaR information. If the VaR results directly serve to compute capital reserves, then the choice for a confidence level is crucial since it reveals the bank's degree of risk aversion, has direct implications on the capital reserves costs, and the costs of excessive losses.

A high degree of risk aversion implies a high level of capital reserves capable of absorbing high potential losses, and this will lead to the selection of a high confidence level. On the other hand, if VaR measures serve to compare or assess various sources of risk across different markets, then the choice of a confidence level is clearly less of an issue.

5.2.2 Portfolio's VaR

VaR provides users with a rapid risk measure of their portfolio with respect to normal market conditions, i.e. notwithstanding unforeseen catastrophes. This number summarises a portfolio's exposure to market risk accompanied with a probability of loss. The following series of tables provide various VaRs for different holding periods at five different confidence levels 95%, 97.5%, 99% and 99.5%. To be able to compute the VaR for horizons smaller than one year, the cash flows periodicity is fixed to 3 months. The figures are computed using the cumulative distribution function presented below for horizon length prices. The excess distribution function is presented for a three month horizon in *Figure 15*.

Confidence level	95%	97.5%	99%	99.5%
T = 1 quarter	1.144	1.996	2.947	3.562
T = 2 quarters	0	0.815	2.172	2.977
T = 3 quarters	0	0	0.794	1.997
T = 1 year	0	0	0	0.123
T = 2 years	0	0	0	0

Table 9: Values at Risk for as function of Time Horizon (initial level = 100)

These results show that for a holding period of two years or longer, the risk of loss can be considered as null for the portfolio in our example.

Conclusion

In this paper, we have considered two methods to give a price to real estate portfolio. Our aim was to show in what way Monte Carlo simulation method could be useful for the measurement of complex cash generating assets such as real estate assets return distribution.

Based on a portfolio example, we have shown that simulated cash flows

- provide more robust valuations than traditional DCF valuations,
- permit the user to estimate the portfolio's price distribution for any time horizon,
- facilitate Values-at-Risk (VaR) computations.

However, if the simulation method drives to more robust results and can measure better the risk for the investor, it is necessary to underline it lies on a sufficient knowledge of the statistics laws that governs the principal variables included in the cash-flows.

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APPENDIX

	<i>DCF</i>	<i>Simulations</i>
initial price P_0 :	(P_0)	$P_0 = 100$
initial rent $Rent_0$:	$Rent_0 = 100/11$	$Rent_0 = 100/11$
initial potential current expenses Exp_0	$Rent_0/6 = 100/66$	$Rent_0/6 = 100/66$
potential rent evolution $Rent_t^*$	$Rent_0 \times (1.063)^t$	$Rent_0 \times \exp(0.0606t + 0.0288W_t^R)$
rent evolution $Rent_t$	$0.85 \times Rent_0 \times (1.063)^t$	$\eta_t \times Rent_t^*$
current expenses evolution Exp_t^*	$Exp_0 \times (1.03)^t$	$Exp_0 \times (1.03)^t$
prices evolution P_t	-	$P_0 \times \exp(0.0678t + 0.0538W_t^P)$
free cash flows at $t < T$, FCF_t	$0.85 \times Rent_0 \times (1.063)^t - Exp_0 \times (1.03)^t$	$Esp(\eta_t \times Rent_t^*) - Exp_0 \times (1.03)^t$
Terminal value	$TV = \frac{FCF_T (1.03)}{0.084 - 0.03}$	$Esp(P_t)$

FIGURES

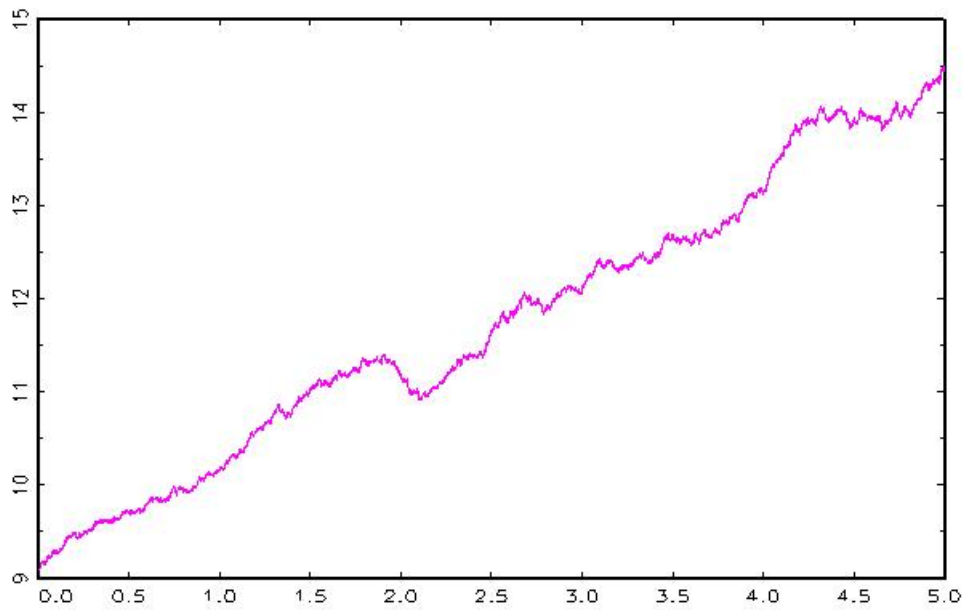


Figure 1: One path for the Brownian process $dS_t/S_t = 0.06 dt + 0.03dW_t$ for 5120 increments

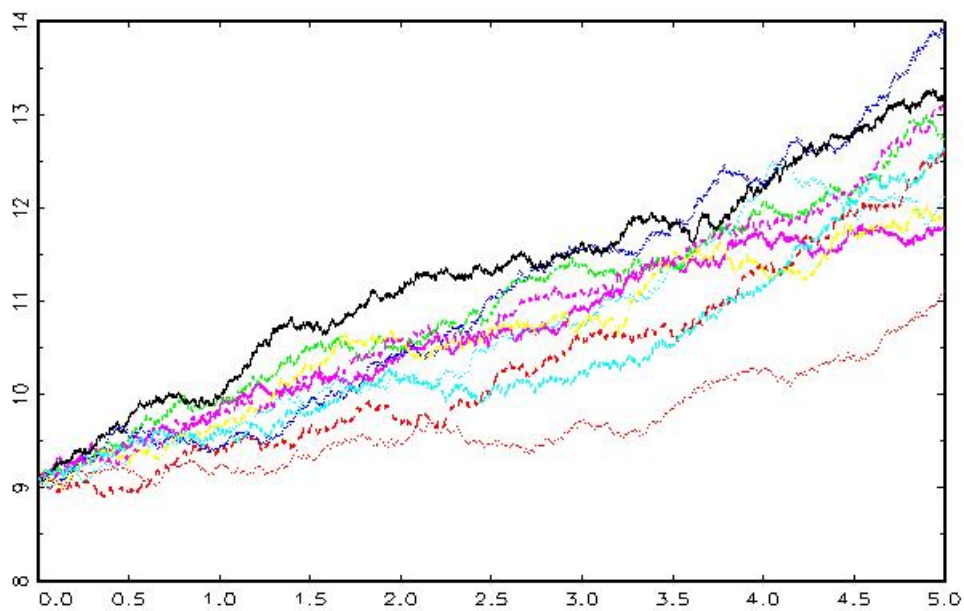


Figure 2: 10 simulations of $dS_t/S_t = 0.06 dt + 0.03dW_t$, with $T = 5$ years

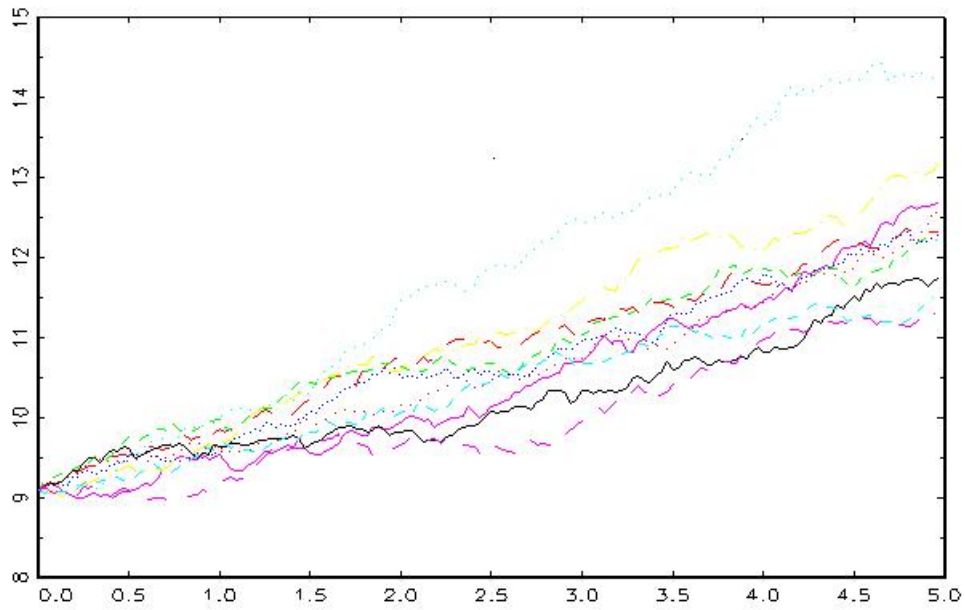


Figure 3: 10 simulations of $d \text{Rent}_t^* / \text{Rent}_t^* = 0.0611dt + 0.0288dW_t^R$ for 5 years

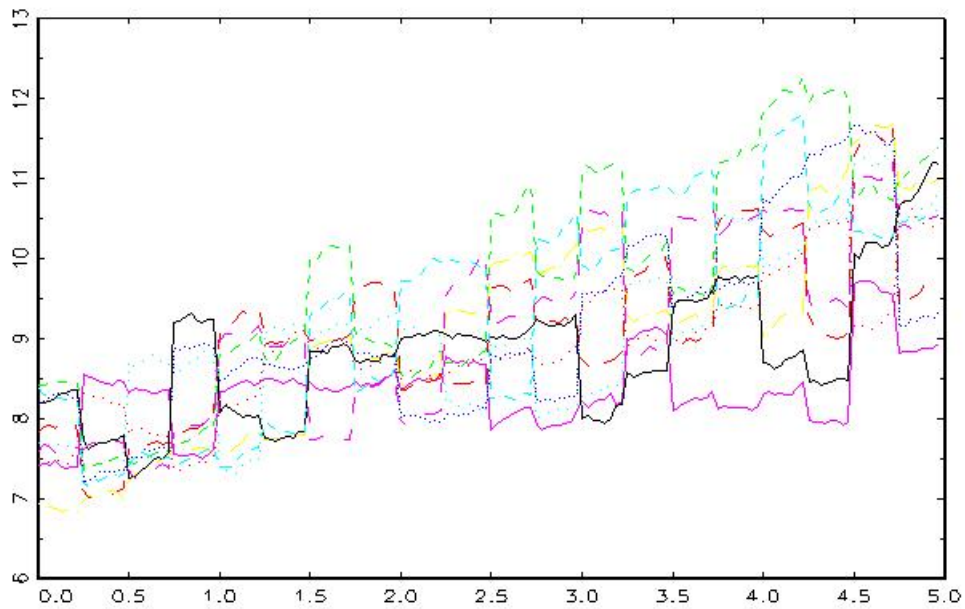


Figure 4: 10 simulated paths of rents combining to occupation rate with a 3 months evolution

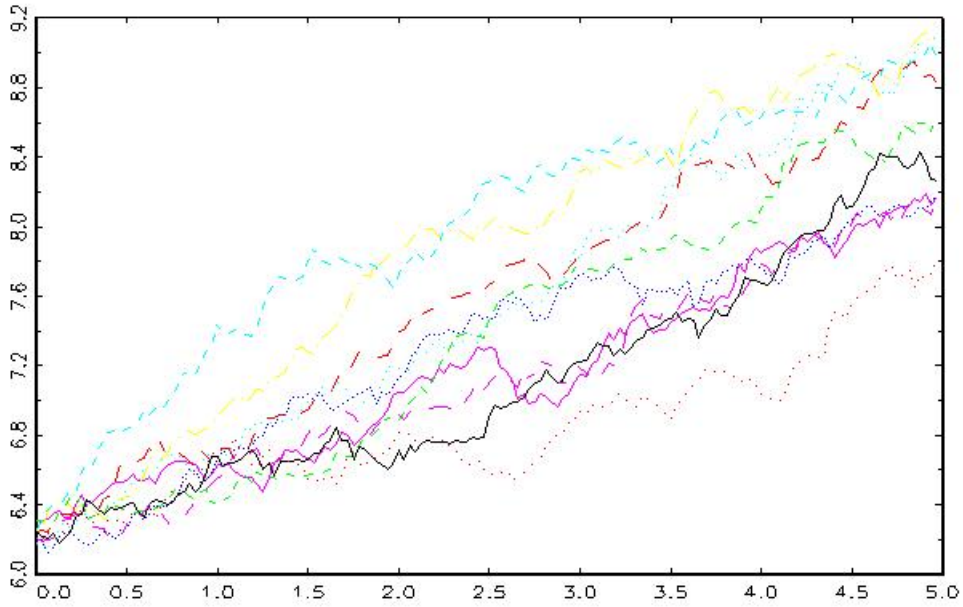


Figure 5: 10 simulated paths for rents after current expenses (occupancy rate = 1)

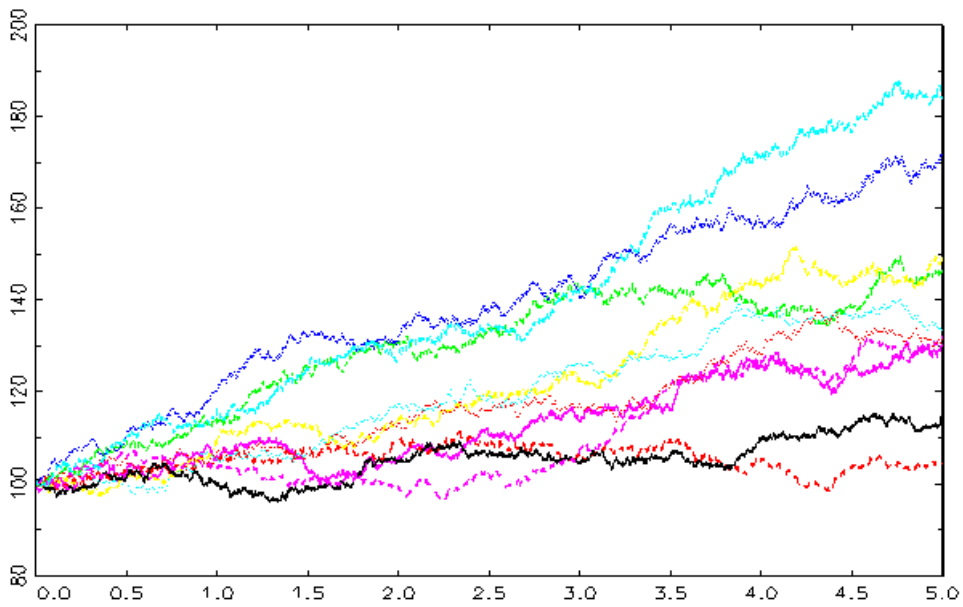


Figure 6: 10 simulated paths of $dP_t / P_t = 0.0692 dt + 0.0538 dW_t^P$

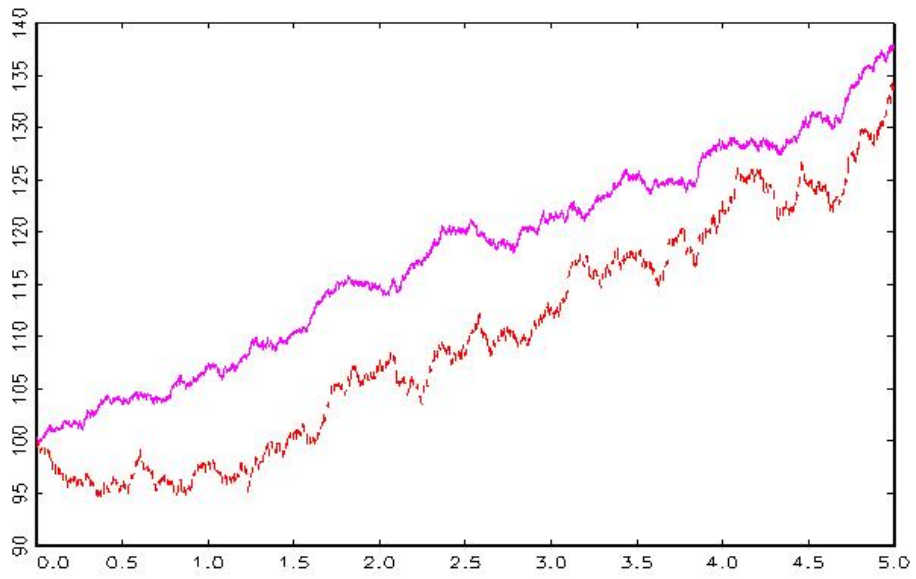


Figure 7: one path of rent (upper curve) and price (lower curve) dynamics (over 5 years)

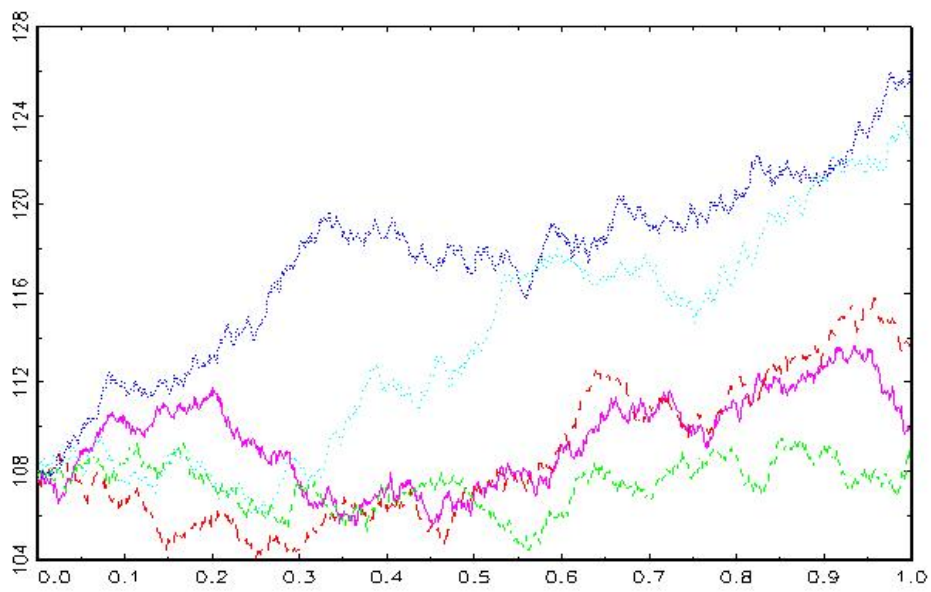


Figure 8: Portfolio potential price dynamics $T = 1$

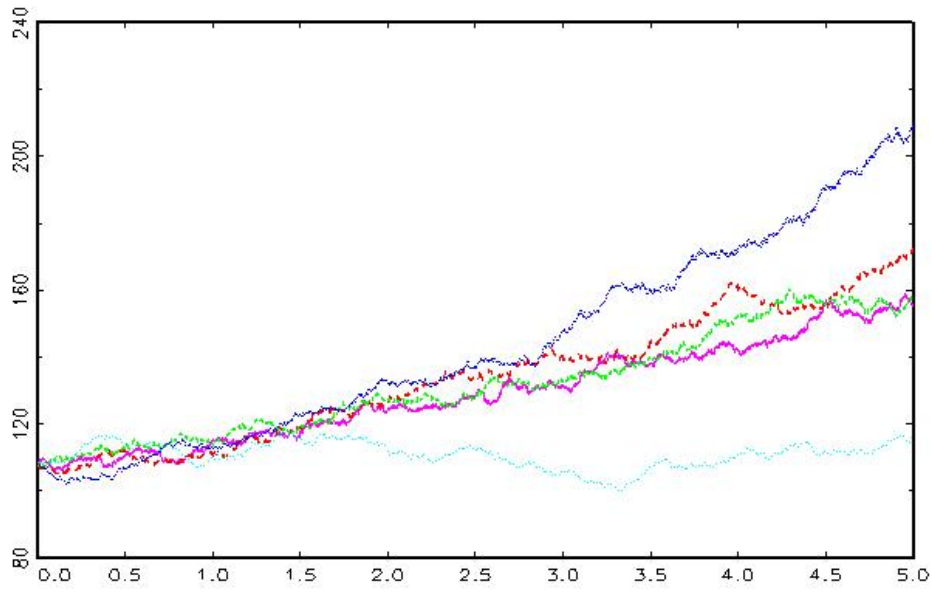


Figure 9: portfolio potential price dynamics $T = 5$

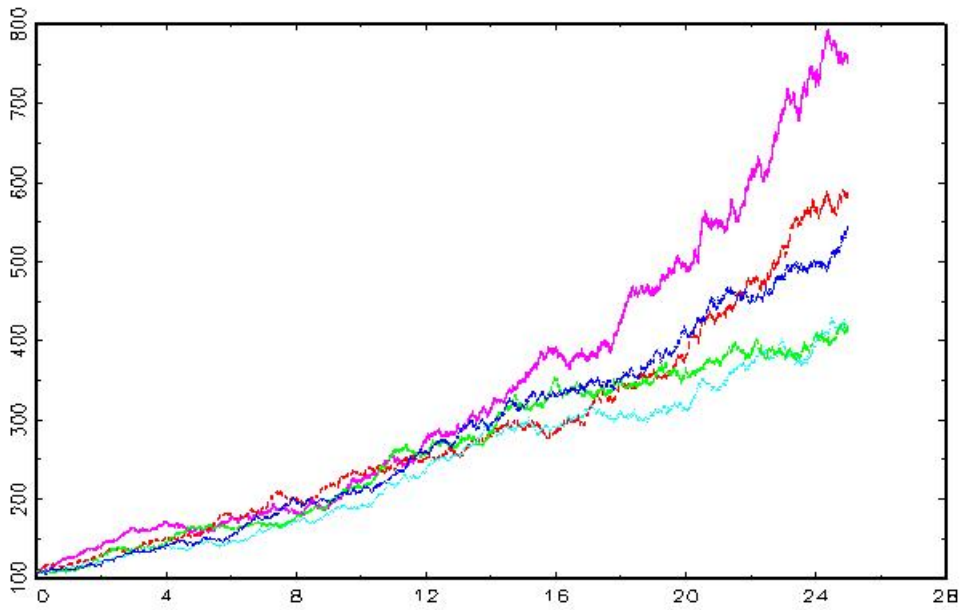
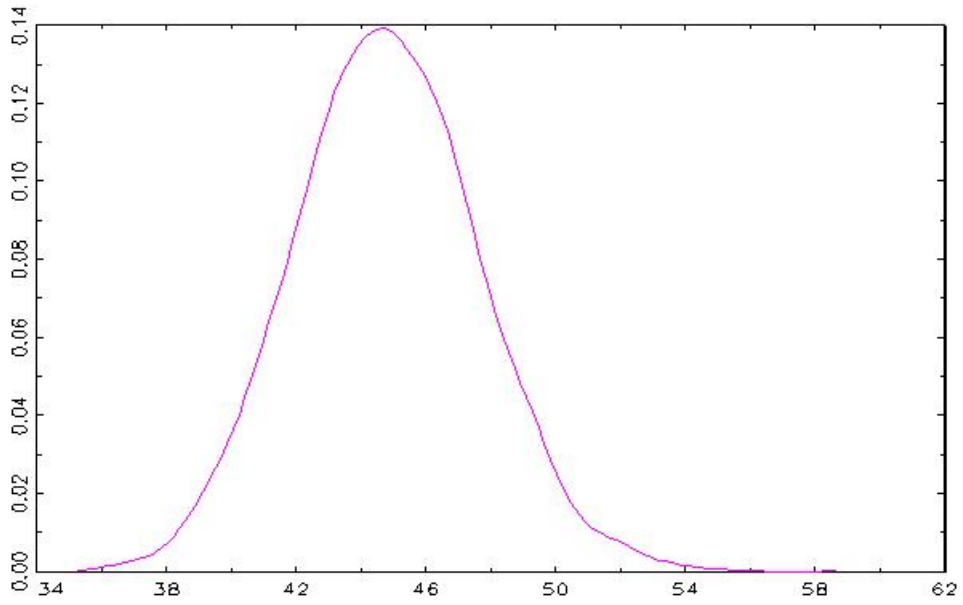


Figure 10: portfolio potential price dynamics $T = 25$



*Figure 11: density of the free cash flows (without TV) at time $T = 5$ years
(sum of the five annual free cash flows valued at time $T = 5$)*

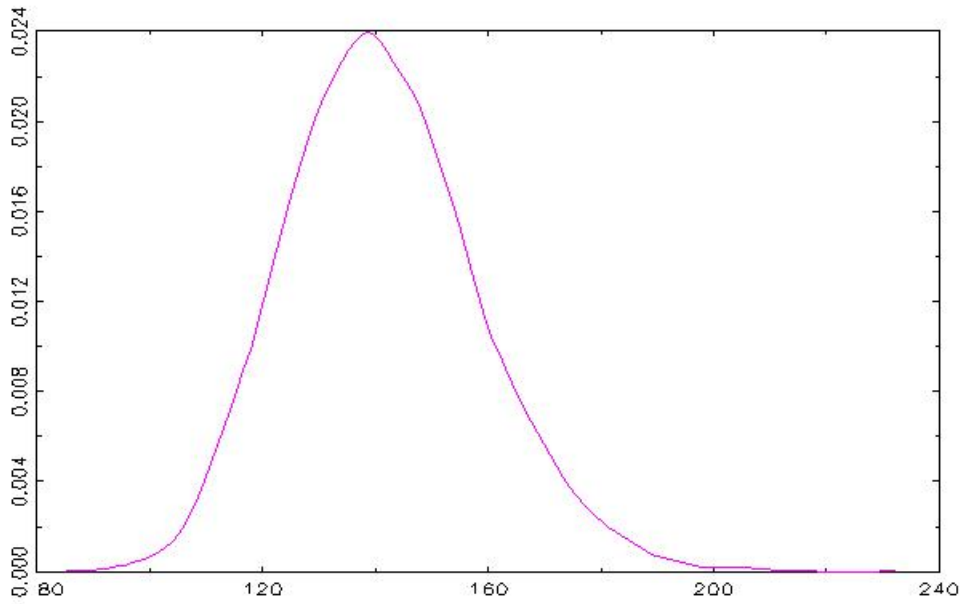


Figure 12: density of the terminal value at time $T = 5$ years

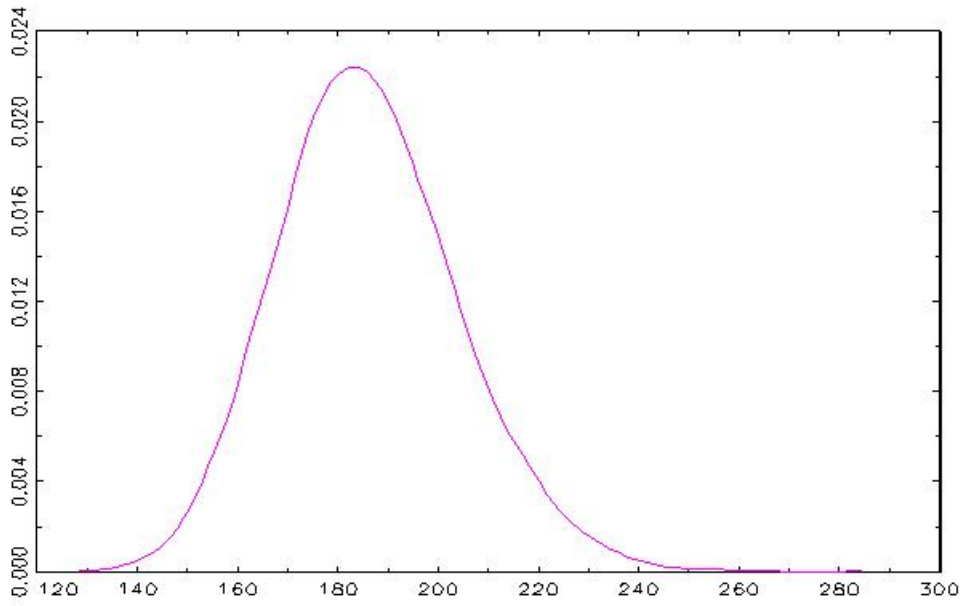


Figure 13: density of the portfolio value at time $T = 5$ years

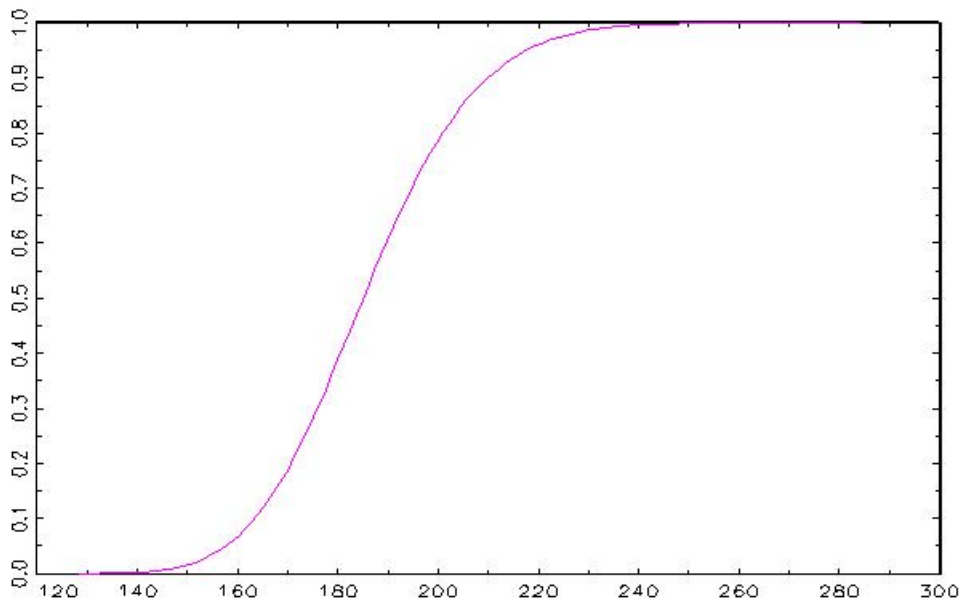


Figure 14: distribution function of the portfolio value at time $T = 5$ years

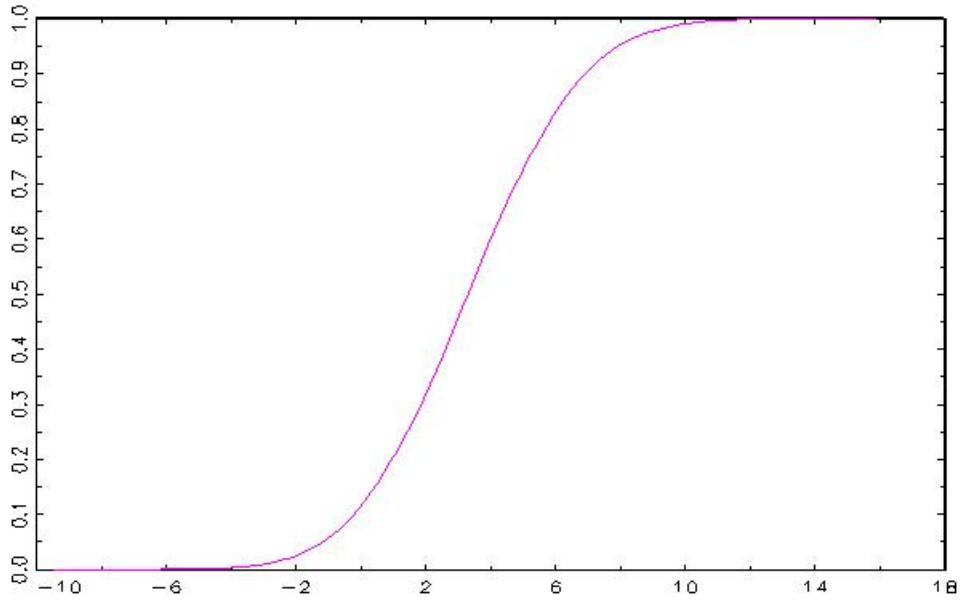


Figure 15: excess to initial price ($P_0 = 100$) distribution function of the portfolio value at time $T = 1$ quarter

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